Economic Growth and Labor Market Institutions in East Asian Structural Transformation*

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Abstract

This paper develops a new growth model by incorporating labor market friction and human capital accumulation into the multi-sector growth framework to analyze the underlying link between economic growth and labor market institutions in a transitional economy. The model, calibrated based on the Japanese and South Korean structural transformation episodes, demonstrates that lifetime employment (and the implied lengthy job tenure) has contributed to endogenous formation of a Ricardian comparative advantage in non-agricultural sector, by enhancing specific human capital accumulation and facilitating investment. It has enabled Japan and South Korea to achieve unprecedentedly rapid economic growth. The counterfactual experiment finds that had the job durations of a typical worker been 1 year (roughly one tenth of the actual average job duration) for 1960-1990 in the Japanese labor market and 1970-2000 in the Korean labor market, the non-agricultural GDP per capita in 1990 would have accounted for 71 and 76 percent of the actual values, respectively.

Keywords: Structural Transformation, Economic Growth, Labor Market Institution
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1 Introduction

There is extensive and still growing interest in non-stationary multi-sector economic growth, or shortly structural transformation. However, since most of the previous studies on structural transformation presume that it is the evolution of Walrasian equilibria guided by the exogenous shock of total factor productivity (hereafter TFP), they are incapable of answering the question, "What enables some countries to perform rapid structural transformation and others not?" Motivated by Mortensen and Pissarides (1994, 1998), this paper develops a new growth model that incorporates labor market friction and human capital accumulation into the multi-sector growth theory to analyze how different labor market institutions contribute differently to economic growth. In particular, our quantitative assessment based on East Asian structural transformation episodes reveals that their lifetime employment system characterized by lengthy job tenures has enabled postwar Japan and South Korea, by contributing to formation of Ricardian comparative advantage in sectors in which it had not previously been experienced, to achieve unprecedentedly rapid structural transformation.

In their seminal work, Aghion and Howitt (1994) show that fast economic growth may reduce unemployment (capitalization effect) or add fuel to unemployment (creative destruction effect). Mortensen and Pissarides (1998) subsequently document the rapid progress of disembodied technology to decrease, and of embodied technology to increase, the steady state unemployment rate. By extending the aforementioned model, Pissarides and Vallanti (2007) and Miyamoto and Takahashi (2011) also evaluate the effect of technological progress on the labor market. All those pioneering models, focusing only on the one-sided effect of technological progress on steady state unemployment, are somewhat far from the neoclassical growth models in which the feedback effect of human capital accumulation in the labor market on economic growth is a central issue. Chen, Chen, and Wang (2011) recently constructed a search model with endogenous human capital accumulation and labor market participation to evaluate the effectiveness of human capital policies. But their framework is not widely applicable to various environments of interest in the growth literature given that they solve only for the one-sector[1] steady state model in which all households, regardless of sector and employment status, share the same rate of human capital accumulation.

Following Matsuyama (1992), we construct an endogenous growth model of a small open economy with agricultural and non-agricultural sectors populated by a continuum of entrepreneurs and workers who consume both agricultural and non-agricultural products. Entrepreneurs employ capital and labor input to produce products in their respective sectors. Each sectoral labor market is subject to search and matching friction, as in Mortensen and Pissarides (1994). Employed workers provide labor and earn wages in the respective sectors, according to the Stole and Zwiebel (1996) bargaining rule, and accumulate job-specific human capital through learning-by-doing on the job. Unemployed workers collect unemployment benefits and search for jobs. They switch to the other sector if the opportunity cost of staying dominates the value of unemployment in their own sector. We assume the population and labor-augmenting technology (general human capital) of workers to grow at a constant rate, technology and capital stock endogenously through investment by forward-looking entrepreneurs. Analyzing

[1] More precisely, it means "one productive sector." Their model consists of two sectors, a productive sector and non-productive sector.

the transition path from initial states to balanced growth paths under rational expectation enables us to investigate rich counterfactual scenarios. To solve for the entire transition path, we iterate the forward- and backward-shooting algorithms following Lipton, Poterba, Sachs, and Summers (1982) and Ishimaru, Oh, and Sim (2013).

We calibrate the model using the structural transformation episode of Japan and South Korea. In addition to Japan, so called Newly Industrializing Countries (NICs) of East Asia, such as Hong Kong, Singapore, South Korea, and Taiwan, have experienced rapid economic growth by transforming from agricultural to non-agricultural economies through specialization in heavy chemical industries. It is an interesting question why only those NICs and Japan were able to form Ricardian comparative advantages on those heavy chemical industries. At least part of the answer likely lies in interaction between specific human capital and the "lifetime employment system" that resulted from the Confucian tradition which ethically discouraged job turnover by employees and dismissal by employers.[2] Although general human capital contributes growth of the economy, it hardly creates or reinforces structural transformation toward a particular sector. In contrast, specific human capital acquired and utilized in a particular sector or firm can be a key driving force of endogenous formation of Ricardian comparative advantage. More precisely, the interaction between specific human capital and lifetime employment system has little effect on the agricultural economy, which does not rely on significant (physical or human) capital accumulation. Rather it accelerates and intensifies specialization toward the non-agricultural sectors that require skilled workers and physical investment on complicated facilities, such as the electronics, automobiles, and heavy-chemical industries.[3] In light of this, the main goal of our calibration is to quantify the social returns to specific human capital and highlight the channel through which the lifetime employment system enhances economic growth.

The counterfactual experiment finds that had the job durations of a typical worker been 1 year (roughly one tenth of the actual average job duration) for 1960-1990 in the Japanese labor market and 1970-2000 in the Korean labor market, the non-agricultural employment share in 1990 would have accounted for 80-85 percent of their actual values and the non-agricultural GDP per capita in 1990 for 71 and 76 percent of their actual values, respectively, suggesting sluggish structural transformation. Also, if the Japanese and South Korean labor markets had transplanted from the flexible U.S. labor market[4] the matching technology and high separation rate, non-agricultural GDP per capita would have been lower by 5-8 percent in both countries, whereas agricultural GDP per capita would have not been affected significantly. It suggests that at least during the period of structural transformation from agricultural to non-agricultural,

[2]According to (OECD) Employment Outlook 1993 (http://www.oecd.org/els/oecdemploymentoutlook-downloadableeditions1989-2011.htm), average (median) job tenure for male workers in Japan, the United Kingdom, and the United States was 12.5 (10.1), 9.2 (5.3), and 7.5 (3.5) years, respectively. The figures imply even shorter job durations for the majority of U.K. and U.S. workers. Esteban-Pretel and Fujimoto (2012) report the quarterly job separation rate in Japan at around 0.02. Chang, Nam, and Rhee (2004) report that of South Korea at between 0.02 and 0.03, roughly one quarter of the U.S. separation rate of 0.1 reported in Shimer (2005).

[3]Davidson, Martin, and Matusz (1999) argue that the labor market institution can determine comparative advantage as well, showing by means of the two-country, two-sector, two-factor trade model that a country with better matching efficiency has comparative advantage over a sector with a higher rate of separation.

[4]Precisely speaking, the matching technology used and calibrated in other studies on the U.S. labor market

the stable labor markets can perform better than flexible ones.

This paper adds to the extant literature on structural transformation several distinctive features. First, it introduces imperfect mobility of productive resources, that is, labor. Previous papers on structural transformation including Hansen and Prescott (2002), Ngai and Pissarides (2007), Duarte and Restuccia (2010), and Buera and Kaboski (2012) assume perfectly competitive labor markets absent any rigidity in factor mobility. Departing from the "immediate full employment assumption," we borrow the concepts of both search friction from Mortensen and Pissarides (1994, 1998) and an inter-sectoral labor barrier from Hayashi and Prescott (2008).[5] The current paper analyzes the dynamic path of non-stationary economic growth impeded by intra- and inter-sectoral labor rigidity as in Ishimaru, Oh, and Sim (2013). Intra-sectoral rigidity (search friction) is introduced to incorporate the effect of job security and job tenure on human capital accumulation and inter-sectoral rigidity (inter-sectoral barrier) to control the direction and duration of the non-stationary structural transformation.

Second, it incorporates both general and specific human capital. General human capital represents the accumulated knowledge or general equipment of the economy. Although not many workers in 1960 knew how to use automobiles, computers, or even telephones, most workers in developed countries now take advantage of cars, computers and cell-phones. These are captured by labor augmenting technology (general human capital) in our model. It works as a permanent TFP shock in other papers analyzing a balanced growth path with a permanent TFP shock. In addition to general human capital, workers individually acquire skills through learning-by-doing in many workplaces. The acquired skills through repetition of similar tasks may be specific to the occupation, job, or working environment. We capture it as job specific human capital of individual workers. Our numerical exercises quantify the effect of general human capital on the overall growth of the economy and that of specific human capital on the unbalanced growth of the sectors, structural transformation.

Third, rather than assuming an exogenously embedded TFP shock, the present paper demonstrates the endogenous formation of a Ricardian comparative advantage. Hayashi and Prescott (2008), Esteban-Pretel and Sawada (2009), and Uy, Yi, and Zhang (2013) analyze the structural transformation episode of Japan and South Korea.[6] In their models, however, the main driving force of structural transformation is the exogenously embedded sectoral TFP shock. Unless their sources are clearly identified, there is a risk of overstating the impact of exogenous sectoral TFP shocks on economic growth. In sharp contrast, this paper, by replacing "perfect foresight" with "rational expectation," develops a non-stationary endogenous growth model in which the dynamic path of the transitional economy is endogenously determined by forward-looking decisions of contemporary economic agents. Apparently, invoking rational expectation enables us to conduct rigorous counterfactual experiments.

The remainder of the paper is organized as follows. We develop the model in section 2 and, present numerical analysis using the Japanese and South Korean episodes in section 3. Based on the calibration in section 3, we conduct counterfactual experiments in section 4, and conclude in section 5.

[5] Hayashi and Prescott (2008) show that the labor barrier in their neoclassical two-sector growth model depressed economic growth in the prewar Japanese economy.

[6] The former allows time-varying sectoral TFP shocks, while the latter constant sectoral TFP shocks.

2 The Baseline Model

2.1 Environment

Consider a small open economy populated by continuum of entrepreneurs and workers who consume both agricultural and non-agricultural products. Entrepreneurs in both sectors, denoted by subscript $i \in \{a, m\}$, manage their own firms and take profit flow π_t at every instant $t \in [0, \infty)$. Workers are either employed or unemployed, the latter in both sectors receiving the same unemployment insurance, b_t, at every instant, the employed workers in sector i earning wage w_{ijt} at time t, depending on skill $j \in \{h, l\}$. The labor market is assumed to be subject to search and matching friction following the Diamond-Mortensen-Pissarides model. Time is continuous and all economic agents discount the future at rate r.

Workers At every instant, each individual worker having income flow $\overline{w}_t$ chooses the consumption bundle (c_{at}, c_{mt}) to maximize

$$(c_{at}^{\frac{\sigma-1}{\sigma}} + c_{mt}^{\frac{\sigma-1}{\sigma}})^{\frac{\sigma}{\sigma-1}}, \tag{1}$$

subject to the budget constraint $p_a c_{at} + p_m c_{mt} = \overline{w}_t$, where p_a and p_m represent the world prices of agricultural and non-agricultural products, respectively. It is assumed that workers are allowed neither to save nor to borrow. Since we are looking at a small open economy, we assume that (p_a, p_m) are exogenously given and constant over time.[7] In particular, we take the non-agricultural products as *numéraire* and normalize p_m to be unity. Note that the elasticity of substitution should be strictly positive, $\sigma > 0$.[8] Solving the static utility maximization problem and plugging the individual demand for each product into the direct utility flow yields the indirect utility flow, as follows.

$$\nu(p_a, p_m, \overline{w}_t) = \overline{w}_t P^{-1} = \nu(\overline{w}_t), \quad \text{where} \quad P = \left(p_a^{1-\sigma} + p_m^{1-\sigma}\right)^{\frac{1}{1-\sigma}}. \tag{2}$$

Denote by L_t the total population of workers at time $t \in [0, \infty)$. At every instant, ρL_t measure of workers retire and χL_t measure of newly born workers enter the labor market as unemployed, which implies that the total population grows at rate $\chi - \rho (\geq 0)$ at every instant. Hence,

$$L_t = e^{(\chi-\rho)t} L_0 \quad \text{at each} \quad t \in [0, \infty). \tag{3}$$

The sectoral ratio of newly-born workers is *ex ante* assumed to be the same as that of existing workers at every instant. But the new workers in one sector can immediately switch to the other sector at switching cost $s \sim Logistic(0, \varepsilon_t/\omega)$. For simplicity, the location parameter of the logistic distribution is normalized to be zero. The scale parameter of the logistic distribution is composed of parameter ω and the labor-augmenting technology ε_t (The role of the labor-augmenting technology will be explained later).

[7] Matsuyama (1992) shows that agricultural productivity is positively (negatively) correlated with economic growth in a closed (small open) economy.

[8] When $\sigma \to 0, 1$, and ∞, the utility function in (1) converges to a Leontief function, Cobb-Douglas function, and von-Neumann function, respectively. Refer to Arrow, Chenery, Minhas, and Solow (1961).

Following Mcfadden (1974), Rust (1987), and Kennan and Walker (2011), we obtain the probability that the newly-born unemployed worker in sector i switches to sector $-i$ as follows.

$$\omega_{-it} = \frac{1}{1 + \exp[-(U_{-it} - U_{it})\omega/\varepsilon_t]} = 1 - \omega_{it}, \tag{4}$$

where U_{it} represents the lifetime value of the unemployed worker in sector i at time t.[9] Unemployed workers retire at rate ρ and receive job offers at rate $f(\theta_{it})$[10]. If employed, the worker enjoys the lifetime value of W_{ilt}. Unemployed workers in sector i also get revision shocks at rate ξ and switch to the other sector if the net gains from doing so is positive. The Hamilton-Jacobi-Bellman (HJB hereafter) equation for the worker is given by

$$rU_{it} = \nu(b_t) - \rho U_{it} + f(\theta_{it})(W_{ilt} - U_{it}) + \xi\mathbb{E}[\max\{U_{-it} - U_{it} - s, 0\}] + \dot{U}_{it}, \tag{5}$$

where

$$\mathbb{E}[\max\{U_{-it} - U_{it} - s, 0\}] = \omega^{-1}\varepsilon_t \log(1 + \exp[(U_{-it} - U_{it})\omega/\varepsilon_t]) =: \Delta_{it} \tag{6}$$

The detailed derivation of (6) is presented as well in Ishimaru, Oh, and Sim (2013). The left-hand side of (5) can be interpreted as the opportunity cost of holding the asset, unemployment at time t. The terms on the right-hand side represent the dividend flow from holding the asset U_{it}, the potential loss from retirement, the potential gains from job finding, the potential gains from inter-sectoral migration, and the gains from changes in valuation of the asset, respectively. To ensure the existence of the balanced growth path, we assume, following Mortensen and Pissarides (1998), that $b_t = b\varepsilon_t$. An unskilled worker employed in sector i receives wage flow w_{ilt} per instant. The worker is separated from the job and becomes unemployed at rate δ and acquires skills at rate ζ. The HJB equations for the skilled and unskilled worker in sector i are, respectively,

$$rW_{iht} = \nu(w_{iht}) - \rho W_{iht} + \delta(U_{it} - W_{iht}) + \dot{W}_{iht}, \quad \text{and} \tag{7}$$

$$rW_{ilt} = \nu(w_{ilt}) - \rho W_{ilt} + \delta(U_{it} - W_{ilt}) + \zeta(W_{iht} - W_{ilt}) + \dot{W}_{ilt}. \tag{8}$$

The left-hand sides of (7) and (8) represent the opportunity cost of holding asset W_{iht} and W_{ilt}, respectively. The right hand side in equation (7) consists of the dividend flow from the asset, the potential loss from retirement, the loss from job separation, and the gains from changes in valuation of the asset, respectively. The right hand side of (8) has one additional term, gains from skill acquisition. It is assumed that "skill" is job-specific so that skilled workers as well as unskilled workers lose their skill and get U_{it} when they are separated from their job. Alternatively, one can think of a variant with "general skills", which requires to extend the dimension of the model. The alternative may cause quantitative changes in our argument, but the qualitative implication of this paper will be maintained. Furthermore, in Japan, South Korea,

[9] Given random switching cost s, the worker in sector i switches to sector $-i$ if and only if $U_{it} < U_{-it} - s$.

$$Pr\{U_{it} < U_{-it} - s\} = \frac{1}{1 + \exp[-(U_{-it} - U_{it})\omega/\varepsilon_t]}.$$

[10] It will be discussed later.

and other East Asian countries having the Confucianism tradition, it is not common for an ordinary worker in one firm to switch to another rival firm and utilize the skills that she/he acquired in the former job. Once she/he is separated from a job, the worker is more likely to get the next job in a different occupation through long unemployment periods. Hence, this paper keeps the specific skills.

Entrepreneurs There are n_i-measure of entrepreneurs in sector $i \in \{a, m\}$, who share the same preferences with workers. Entrepreneurs in each sector produce homogeneous products using identical production technology, given by

$$y_{it} = a_{it}^{\beta_{ai}} k_{it}^{\beta_{ki}} (\varepsilon_t \tilde{l}_{it})^{\beta_{li}}, \tag{9}$$

where $(a_{it}, k_{it}, \tilde{l}_{it})$ represent the TFP component, capital stock, and labor input at time t, respectively. It is assumed that $\beta_{ai} + \beta_{ki} + \beta_{li} = 1$. The labor-augmenting technology ε_t is introduced to capture the growth of the accumulated (general) knowledge of the economy. It can be interpreted as general human capital. Sustained economic growth on the balanced growth path is obtained by assuming ε_t to grow permanently, such that

$$\dot{\varepsilon}_t = \psi \varepsilon_t \quad \text{for each} \quad t \in [0, \infty). \tag{10}$$

The total factor productivity component, a_{it}, can be formed gradually through R&D investment and interaction with other productive resources (*i.e.* human capital). Throughout the paper we call it 'technology'. That many R&D outcomes including new technologies and equipment can be utilized together with a certain level of skills accounts for gradual and persistent economic growth in a transitional economy. We posit

$$\dot{a}_{it} = -\eta_a a_{it} + \lambda_i z_{it}^{\kappa_i} (\varepsilon_t L_{iht})^{1-\kappa_i}, \tag{11}$$

where λ_i is the efficiency parameter of technology investment, κ_i is the elasticity of technology investment, and η_a is the technology deterioration rate. L_{iht}, L_{ilt}, and L_{it} are the number of the skilled employed, unskilled employed, and all workers, respectively, in sector i at time t. Each entrepreneur in sector i makes R&D investment of z_{it} at every t. The law of motion in (11) implies that that R&D investment becomes more efficient as the skilled population in sector i increases (human capital externality).[11] This, together with deterioration, embodies the gradual formation of the Ricardian comparative advantage and disadvantage. The capital stock, which can be immediately accumulated by purchasing from the international capital market, is depreciated at rate η_k.

$$\dot{k}_{it} = -\eta_k k_{it} + x_{it}, \tag{12}$$

where x_{it} is the capital investment in sector i at time t. Regarding the labor input, it is assumed that

$$\tilde{l}_{it} = l_{ilt} + \alpha_i l_{iht}, \tag{13}$$

[11] This positive externality is motivated by Lucas (1988) and Choi (2011). By calibrating the model and conducting a counterfactual analysis, Choi (2011) argues that human capital externality has a significant effect on economic growth. Romer (1986, 1987) proposed the growth models with physical capital externalities in advance.

where (l_{ilt}, l_{iht}) represent the masses of unskilled and skilled workers, respectively, employed by the entrepreneur at time t. The coefficient α_i reflects the fact that a skilled worker produces $\alpha_i (\geq 1)$ times more than an unskilled worker. Again, "skill" is assumed to be job-specific. Let v_{it} be the number of vacancies waiting for unemployed workers. The entrepreneur finds at rate $q(\theta_{it})$ a worker who starts producing as an unskilled worker. Unskilled workers get learning shocks at rate ζ on the job and become skilled workers. All workers leave the entrepreneur due to separation shock at rate δ and retirement shock at rate ρ. The law of motion of the employed workers is described by

$$\dot{l}_{iht} = -(\delta + \rho) l_{iht} + \zeta l_{ilt}, \quad \text{and} \tag{14}$$

$$\dot{l}_{ilt} = -(\delta + \rho + \zeta) l_{ilt} + q(\theta_{it}) v_{it}. \tag{15}$$

The profit flow of the entrepreneur in sector i at time t is given by

$$\pi_{it} = p_i y_{it} - \sum_{j=h,l} w_{ijt} l_{ijt} - p_k x_{it} - p_z z_{it} - \gamma \varepsilon_t v_{it}, \tag{16}$$

where (p_k, p_z) represent the cost of capital investment and R&D investment, respectively, and $\gamma \varepsilon_t$ represents the cost of creating a vacancy. Following Mortensen and Pissarides (1998), we assume the vacancy creation cost to grow together with ε_t, which is necessary to ensure the existence of the balanced growth path. The entrepreneur in sector i having $(\bar{\varepsilon}, \bar{a}_i, \bar{k}_i, \bar{l}_{hi}, \bar{l}_{li})$ at time t chooses the schedule of $(x_{i\tau}, v_{i\tau}, z_{i\tau})$ for each $\tau \in [t, \infty)$ to maximize

$$E_{it}(\bar{\varepsilon}, \bar{a}_i, \bar{k}_i, \bar{l}_{hi}, \bar{l}_{li}) = \max_{z_{i\tau}, x_{i\tau}, v_{i\tau} \geq 0} \int_t^\infty e^{-r(\tau - t)} \pi_{i\tau} P^{-1} d\tau \tag{17}$$

subject to (10), (11), (12), (14), (15) and the initial condition $(\varepsilon_t, a_{it}, k_{it}, l_{iht}, l_{ilt}) = (\bar{\varepsilon}, \bar{a}_i, \bar{k}_i, \bar{l}_{hi}, \bar{l}_{li})$. Lemma 1 solves the optimal control problem by the entrepreneur.

Lemma 1 *The entrepreneur in sector $i \in \{a, m\}$ optimally chooses (x_{it}, v_{it}, z_{it}) for each $t \in [0, \infty)$ such that*

$$p_z = \kappa_i \lambda_i z_{it}^{\kappa_i - 1} (\varepsilon_t L_{iht})^{1 - \kappa_i} \int_t^\infty e^{-(r + \eta_a)(\tau - t)} \frac{\partial \pi_{i\tau}}{\partial a_{i\tau}} d\tau \tag{18}$$

$$p_k = \int_t^\infty e^{-(r + \eta_k)(\tau - t)} \frac{\partial \pi_{i\tau}}{\partial k_{i\tau}} d\tau, \quad \textit{and} \tag{19}$$

$$\gamma \varepsilon_t = q(\theta_{it}) \int_t^\infty e^{-(r + \delta + \rho + \zeta)(\tau - t)} \Big[\frac{\partial \pi_{i\tau}}{\partial l_{il\tau}} + \zeta \frac{\partial \pi_{i\tau}}{\partial l_{ih\tau}} \Big] d\tau. \tag{20}$$

In equations (18), (19), and (20), the left-hand side represents the marginal cost of investment, while the right-hand sides represent the marginal benefit. The proof of Lemma 1 is delayed in Appendix A.

Labor Market The degree of the labor market tightness in sector i is defined as the ratio of the measure of total vacancy to that of job seekers in the sector. For each $i \in \{a, m\}$,

$$\theta_{it} := \frac{v_{it} n_i}{u_{it}}. \tag{21}$$

Let $m(v_{it}n_i, u_{it})$ be the number of matches successfully formed at time t for each $i \in \{a, m\}$. Given constant returns to scale matching technology, we obtain

$$f(\theta_{it}) = \frac{m(v_{it}n_i, u_{it})}{u_{it}} = m(\theta_{it}, 1) = \theta_{it} m(1, \theta_{it}^{-1}) = \frac{\theta_{it} m(v_{it}n_i, u_{it})}{v_{it}n_i} = \theta_{it} q(\theta_{it}). \quad (22)$$

We denote by $(L_{iht}, L_{ilt}, u_{it})$ the measure of skilled employees, unskilled employees, and unemployed workers, respectively, in sector i at time t. The total population, L_t, at time t is given by

$$L_t = \sum_{j=h,l} [L_{ajt} + L_{mjt}] + u_{at} + u_{mt} \quad (23)$$

The dynamic worker flows are summarized as follows.

$$\dot{L}_{iht} = -(\rho + \delta) L_{iht} + \zeta L_{ilt}, \quad (24)$$

$$\dot{L}_{ilt} = -(\rho + \delta + \zeta) L_{ilt} + \theta_{it} q(\theta_{it}) u_{it}, \quad \text{and} \quad (25)$$

$$\dot{u}_{it} = -(\rho + \theta_{it} q(\theta_{it}) + \xi \omega_{-it}) u_{it} + \xi \omega_{it} u_{-it} + \delta (L_{ilt} + L_{iht}) + \chi \omega_{it} L_t, \quad (26)$$

where L_t is presented in (3). The last term of the right-hand side of (26) is derived by the following argument: At time t, there are $\chi(L_{aht} + L_{alt} + u_{at})$-measure of newly born workers in the agricultural sector and $\chi(L_{mht} + L_{mlt} + u_{mt})$-measure of newly born workers in the manufacturing sector; because they can immediately switch, the total measure of newly born workers in sector i after the immediate switching is given by $\chi \omega_{it} L_t$.

Wage Bargaining In accordance with Stole and Zwiebel (1996)[12], successfully matched workers and entrepreneurs individually negotiate wages by splitting the marginal surplus. This implies that for each $j \in \{h, l\}$,

$$(1 - \phi)(W_{ijt} - U_{it}) = \phi \frac{\partial E_{it}}{\partial l_{ijt}} \quad \text{at every} \quad t \in [0, \infty), \quad (27)$$

where ϕ is the worker's bargaining power. In wage bargaining, the firm takes $1 - \phi$ portion of the joint (marginal) surplus and gives ϕ portion to the worker in the form of wage payment. Note that (27) is implicitly based on an equilibrium restriction such that in any equilibrium $W_{ijt} - U_{it} \geq 0$ and $\frac{\partial E_{it}}{\partial l_{ijt}} \geq 0$ for each $i \in \{a, m\}$, $j \in \{h, l\}$, and $t \in [0, \infty)$. Since the bargaining rule in (27) is true at every t, continuity and differentiability implies that

$$(1 - \phi)(\dot{W}_{ijt} - \dot{U}_{it}) = \phi \frac{\partial}{\partial t} \Big(\frac{\partial E_{it}}{\partial l_{ijt}} \Big) \quad \text{for almost everywhere} \quad t \in [0, \infty). \quad (28)$$

Lemma 2 *The implied wage in sector $i \in \{a, m\}$ is given by*

$$w_{iht} = A_i^0 \alpha_i p_i \beta_{li} a_{it}^{\beta_{ai}} k_{it}^{\beta_{ki}} \varepsilon_t^{\beta_{li}} (l_{ilt} + \alpha_i l_{iht})^{\beta_{li} - 1} + A_{it}^1, \quad \text{and} \quad (29)$$

[12] Helpman and Itskhoki (2010), Helpman, Itskhoki, and Redding (2010), Felbermayr, Prat, and Schmerer (2011), and Ishimaru, Oh, and Sim (2013) adopt the bargaining rule proposed by Stole and Zwiebel (1996) in their studies on international trade.

$$w_{ilt} \quad = \quad A_i^0 p_i \beta_{li} a_{it}^{\beta_{ai}} k_{it}^{\beta_{ki}} \varepsilon_t^{\beta_{li}} (l_{ilt} + \alpha_i l_{iht})^{\beta_{li}-1} + A_{it}^1, \tag{30}$$

where

$$A_i^0 = \frac{\phi}{1 - \phi + \phi\beta_{li}} \quad \textit{and} \quad A_{it}^1 = (1-\phi)b\varepsilon_t + \gamma\varepsilon_t\phi\theta_{it} + (1-\phi)\xi\Delta_{it}P.$$

Note that in both (29) and (30) the first terms are proportional to the marginal product of labor. The second terms reflect the labor market condition. For later use, we remark that A_{it}^1 grows at rate ψ if θ_{it} is constant over time and Δ_{it} grows at rate ψ. The detailed derivation is provided in Appendix A.

Equilibrium We finish this section by defining the equilibrium of our interest. The following definition summarizes the overall shape of our model.

Definition An *equilibrium* consists of bounded time series of choice rules $\{\omega_{it}, x_{it}, v_{it}, z_{it}\}_{i\in\{a,m\}}$, labor market tightness parameters $\{\theta_{it}\}_{i\in\{a,m\}}$, wages $\{w_{it}\}_{i=\{a,m\}}$, value equations $\{E_{it}, W_{iht}, W_{ilt}, U_{it}\}_{i\in\{a,m\}}$, and laws of motions $\{\varepsilon_t, a_{it}, k_{it}, l_{iht}, l_{ilt}, L_t, L_{iht}, L_{ilt}, u_{it}\}_{i\in\{a,m\}}$ at every $t \in [0,\infty)$ such that:

(*i*) unemployed workers as well as newly born workers optimally choose where they work when they are hit by an exogenous shock,

(*ii*) each entrepreneur in sector i optimally chooses $\{z_{it}, x_{it}, v_{it}\}$ at every t,

(*iii*) aggregate consistency requires that

- the market tightness $\{\theta_{it}\}_{i\in\{a,m\}}$ should be consistent with its definition,
- wages $\{w_{it}\}_{i\in\{a,m\}}$ should be consistent with the imposed bargaining rule,
- $L_{iht} = n_i l_{iht}$, $L_{ilt} = n_i l_{ilt}$, and $L_t = \sum_i (L_{iht} + L_{ilt} + u_{it})$ for each $i \in \{a,m\}$.

(*iv*) the evolution of the entire system is recursively governed by the law of motions of (3), (5), (7), (8), (10), (11), (12), (14), (15),(17), (24), (25), and (26), given $\{E_{i0}, W_{ih0}, W_{il0}, U_{i0}\}_{i\in\{a,m\}}$ and $\{\varepsilon_0, a_{i0}, k_{i0}, l_{ih0}, l_{il0}, L_0, L_{ih0}, L_{il0}, u_{i0}\}_{i\in\{a,m\}}$.

2.2 On the Balanced Growth Path

In this subsection, we characterize the balanced growth paths by applying the previous definition with a small modification on the law of motions (v). Stationarity on the balanced growth path requires that

$$\frac{\dot{L_t}}{L_t} = \frac{\dot{L_{iht}}}{L_{iht}} = \frac{\dot{L_{ilt}}}{L_{ilt}} = \frac{\dot{u_{it}}}{u_{it}} = \chi - \rho, \quad \frac{\dot{a_{it}}}{a_{it}} = \frac{\dot{k_{it}}}{k_{it}} = \chi - \rho + \psi, \quad \text{and}$$
$$\frac{\dot{U}_{it}}{U_{it}} = \frac{\dot{W}_{iht}}{W_{iht}} = \frac{\dot{W}_{ilt}}{W_{ilt}} = \frac{\dot{\varepsilon}_t}{\varepsilon_t} = \psi. \tag{31}$$

Let L_{ih}, L_{il} and u_i denote the proportion of skilled, unskilled and unemployed workers, respectively, in sector i to the total population. That is,

$$L_{ih} = \frac{L_{iht}}{L_t}, \quad L_{il} = \frac{L_{ilt}}{L_t}, \quad \text{and} \quad u_i = \frac{u_{it}}{L_t}. \tag{32}$$

By the stationarity condition of the balanced growth path (31), these ratios are constant over time. The stationarity condition dictates that $\Delta_{it}/\varepsilon_t$ is constant over time on the balanced growth path. Also, from (15), (21), and (31), we get

$$v_{it} = \frac{(\chi+\delta+\zeta)l_{ilt}}{q(\theta_{it})} = \frac{(\chi+\delta+\zeta)L_{ilt}}{q(\theta_{it})n_i} = \frac{\theta_{it}u_{it}}{n_i} \tag{33}$$

on the balanced growth path. Since L_{ilt} and u_{it} grow together at the rate $(\chi-\rho)$, θ_{it} should be constant on the balanced growth path to make the last equality hold over time. We drop the time subscript t from the variables that are constant on the balanced growth path.

Using (31), we rewrite (5) as follows.

$$(r+\rho-\psi)U_{it} \quad = \quad b\varepsilon_t P^{-1} + f(\theta_i)(W_{ilt}-U_{it}) + \xi\Delta_{it}. \tag{34}$$

Combining (20) and (27) yields

$$\gamma\varepsilon_t = Pq(\theta_{it})\frac{\partial E_{it}}{\partial l_{ilt}} = Pq(\theta_{it})\frac{(1-\phi)(W_{ilt}-U_{it})}{\phi}. \tag{35}$$

Subtracting U_{at} from U_{mt}, dividing by ε_t, and combining with (35) yields

$$(U_{mt}-U_{at})/\varepsilon_t = \frac{\gamma\phi(\theta_m-\theta_a)}{(r+\rho-\psi+\xi)P(1-\phi)} \quad \text{and} \tag{36}$$

$$\omega_a = \frac{1}{1+\exp[(U_{mt}-U_{at})\omega/\varepsilon_t]} = 1-\omega_m. \tag{37}$$

These imply that (ω_a, ω_m) and $(U_{mt}-U_{at})/\varepsilon_t$ are constant over time on the balanced growth path and uniquely determined by (θ_a, θ_m). From (25), (26), and (31), the triplet of (L_{ih}, L_{il}, u_i) is characterized as follows. Given (θ_a, θ_m), for each $i \in \{a, m\}$,

$$0 \quad = \quad (\chi+\delta)L_{ih} - \zeta L_{il}, \quad \text{and} \tag{38}$$

$$0 \quad = \quad (\chi+\delta+\zeta)L_{il} - \theta_i q(\theta_i)u_i, \quad \text{and} \tag{39}$$

$$0 \quad = \quad (\chi+\theta_i q(\theta_i)+\xi\omega_{-i})u_i - \xi\omega_i u_{-i} - \delta(L_{il}+L_{ih}) - \chi\omega_i, \tag{40}$$

Solving (37), (39) and (40), and multiplying by L_t results in Lemma 3.

Lemma 3 *Given (θ_a, θ_m), on the balanced growth path,*

$$L_{iht} \quad = \quad \frac{(\chi+\delta)f(\theta_i)\chi\omega_i(\chi+\xi\omega_i+f(\theta_{-i})\chi/(\chi+\delta)+\xi\omega_{-i})L_t}{\zeta(\chi+\delta+\zeta)\{(\chi+\xi\omega_{-i}+\frac{f(\theta_i)\chi}{\chi+\delta})(\chi+\xi\omega_i+\frac{f(\theta_{-i})\chi}{\chi+\delta})-\xi^2\omega_i\omega_{-i}\}}, \tag{41}$$

$$L_{ilt} \quad = \quad \frac{f(\theta_i)\chi\omega_i(\chi+\xi\omega_i+f(\theta_{-i})\chi/(\chi+\delta)+\xi\omega_{-i})L_t}{(\chi+\delta+\zeta)\{(\chi+\xi\omega_{-i}+\frac{f(\theta_i)\chi}{\chi+\delta})(\chi+\xi\omega_i+\frac{f(\theta_{-i})\chi}{\chi+\delta})-\xi^2\omega_i\omega_{-i}\}}, \quad \text{and} \tag{42}$$

$$u_{it} \quad = \quad \frac{\chi\omega_i(\chi+\xi\omega_i+f(\theta_{-i})\chi/(\chi+\delta)+\xi\omega_{-i})L_t}{\{(\chi+\xi\omega_{-i}+\frac{f(\theta_i)\chi}{\chi+\delta})(\chi+\xi\omega_i+\frac{f(\theta_{-i})\chi}{\chi+\delta})-\xi^2\omega_i\omega_{-i}\}} \tag{43}$$

Notice that $\frac{\partial y_{it}}{\partial l_{ilt}}$ grows at rate ψ on the balanced growth path due to the growth of the labor-augmenting technology. Lemma 4 summarizes the entrepreneur's behavior on the balanced growth path.

Lemma 4 *Given* (θ_a, θ_m) *on the balanced growth path, the entrepreneur in each sector optimally chooses*

$$z_{it} = \varepsilon_t L_{iht}\Big[\frac{\lambda_i\kappa_i\beta_{ai}p_i(1-\beta_{li}A_i^0)}{p_z(r+\eta_a)\beta_{li}a_{it}}\frac{\partial y_{it}}{\partial l_{ilt}}(l_{ilt}+\alpha_i l_{iht})\Big]^{\frac{1}{1-\kappa_i}}, \tag{44}$$

$$x_{it} = (\chi-\rho+\psi+\eta_k)\frac{(1-\beta_{li}A_i^0)\beta_{ki}p_i(l_{ilt}+\alpha_i l_{iht})}{\beta_{li}p_k(r+\eta_k)}\frac{\partial y_{it}}{\partial l_{ilt}}, \quad \text{and} \tag{45}$$

$$v_i = \frac{\theta_i\chi\omega_i(\chi+\xi\omega_i+f(\theta_{-i})\chi/(\chi+\delta)+\xi\omega_{-i})L_t}{(\chi+\xi\omega_{-i}+\frac{f(\theta_i)\chi}{\chi+\delta})(\chi+\xi\omega_i+\frac{f(\theta_{-i})\chi}{\chi+\delta})-\xi^2\omega_i\omega_{-i}}, \tag{46}$$

where

$$\frac{\partial y_{it}}{\partial l_{ilt}} = \Big[\beta_{li}\Big(\frac{\lambda_i\varepsilon_t L_{iht}}{\chi-\rho+\psi+\eta_a}\Big)^{\beta_{ai}(1-\kappa_i)}\Big(\frac{\lambda_i\kappa_i\beta_{ai}p_i(1-\beta_{li}A_i^0)}{p_z\beta_{li}(r+\eta_a)}(l_{ilt}+\alpha_i l_{iht})\Big)^{\beta_{ai}\kappa_i}$$
$$\Big(\frac{(1-\beta_{li}A_i^0)\beta_{ki}p_i(l_{ilt}+\alpha_i l_{iht})}{\beta_{li}p_k(r+\eta_k)}\Big)^{\beta_{ki}}\varepsilon_t^{\beta_{li}}(l_{ilt}+\alpha_i l_{iht})^{\beta_{li}-1}\Big]^{\frac{1}{1-\beta_{ki}-\beta_{ai}\kappa_i}}. \tag{47}$$

In addition, these imply that

$$a_{it} = \Big[\frac{\lambda_i\varepsilon_t L_{iht}}{\chi-\rho+\psi+\eta_a}\Big]^{1-\kappa_i}\Big[\frac{\lambda_i\kappa_i\beta_{ai}p_i(1-\beta_{li}A_i^0)}{p_{zi}\beta_{li}(r+\eta_a)}\frac{\partial y_{it}}{\partial l_{ilt}}(l_{ilt}+\alpha_i l_{iht})\Big]^{\kappa_i}, \tag{48}$$

$$k_{it} = \frac{(1-\beta_{li}A_i^0)\beta_{ki}p_i(l_{ilt}+\alpha_i l_{iht})}{\beta_{li}p_k(r+\eta_k)}\frac{\partial y_{it}}{\partial l_{ilt}}, \tag{49}$$

$$l_{iht} = \frac{(\chi+\delta)f(\theta_i)\chi\omega_i(\chi+\xi\omega_i+f(\theta_{-i})\chi/(\chi+\delta)+\xi\omega_{-i})L_t}{n_i\zeta(\chi+\delta+\zeta)\big\{(\chi+\xi\omega_{-i}+\frac{f(\theta_i)\chi}{\chi+\delta})(\chi+\xi\omega_i+\frac{f(\theta_{-i})\chi}{\chi+\delta})-\xi^2\omega_i\omega_{-i}\big\}}, \quad \textit{and} \tag{50}$$

$$l_{ilt} = \frac{f(\theta_i)\chi\omega_i(\chi+\xi\omega_i+f(\theta_{-i})\chi/(\chi+\delta)+\xi\omega_{-i})L_t}{n_i(\chi+\delta+\zeta)\big\{(\chi+\xi\omega_{-i}+\frac{f(\theta_i)\chi}{\chi+\delta})(\chi+\xi\omega_i+\frac{f(\theta_{-i})\chi}{\chi+\delta})-\xi^2\omega_i\omega_{-i}\big\}}. \tag{51}$$

Plugging (48),(49), and (51) into (20) and reordering yields

$$\frac{\gamma}{\beta_{li}q(\theta_{it})}+\frac{A_{it}^1/\varepsilon_t}{(r+\rho+\delta-\psi)\beta_{li}} \tag{52}$$
$$= \Big[\frac{1-\alpha_i}{r+\delta+\rho-\psi+\zeta}+\frac{\alpha_i}{r+\delta+\rho-\psi}\Big]\Big[\Big(\frac{\lambda_i L_{iht}}{\chi-\rho+\psi+\eta_a}\Big)^{\beta_{ai}(1-\kappa_i)}$$
$$\Big(\frac{\lambda_i\kappa_i\beta_{ai}}{p_z(r+\eta_a)}\Big)^{\beta_{ai}\kappa_i}\Big(\frac{\beta_{ki}}{p_k(r+\eta_k)}\Big)^{\beta_{ki}}(1-\beta_{li}A_i^0)p_i(l_{ilt}+\alpha_i l_{iht})^{-\beta_{ai}(1-\kappa_i)}\Big]^{\frac{1}{1-\beta_{ki}-\beta_{ai}\kappa_i}},$$

for each $i \in \{a, m\}$. As mentioned before, (35) is based on the implicit restriction such that $W_{it} - U_{it} > 0$ and $\frac{\partial E_{it}}{\partial l_{it}} > 0$ for each $i \in \{a, m\}$ and $t \in [0, \infty)$. If it is violated, the right-hand side of (52) can be negative so that there is no solution. If the implicit restriction is satisfied, we get two equations described in (52) to solve for two unknowns (θ_a, θ_m).

Proposition 1 *There exists a balanced growth path if and only if the system of equations described in* (52) *has a solution of* (θ_a, θ_m).

Given the complexity of the overall system, it is difficult to analytically determine under what parametric condition the balanced growth path exists and whether it is unique. Instead, we acknowledge that in our numerical experiment with reasonable parameter values, we obtain the same result regardless of the random starting points (we repeat the same experiment with more than 20 different random initial guesses).

2.3 On the Postwar Transition

Here, we characterize the transition path from a particular initial state to the balanced growth path, with the evolution of the economy governed by the system of differential equations. One advantage of our model is that the transition path is fully described by an autonomous control.

The lifetime value of unemployment evolves as follows. For each $i \in \{a, m\}$,

$$\dot{U}_{it} = (r+\rho)U_{it} - \frac{\theta_{it}\gamma\varepsilon_t\phi}{P(1-\phi)} - \xi\Delta_{it} - b\varepsilon_t P^{-1} \quad \text{with} \quad \lim_{t\to\infty} U_{it} = U_i \tag{53}$$

Given $(\theta_{at}, \theta_{mt})$, equation (53) together with (6) determines $(U_{at}, U_{mt}, \Delta_{at}, \Delta_{mt})$ at every $t \in [0, \infty)$. Plugging (53) into (4) also yields $(\omega_{at}, \omega_{mt})$. Starting from $(L_{ah0}, L_{al0}, L_{mh0}, L_{ml0}, u_{a0}, u_{m0})$, $(L_{aht}, L_{alt}, L_{mht}, L_{mlt}, u_{at}, u_{mt})$ evolve following (24)-(26) toward

$$\lim_{t\to\infty}(L_{iht}, L_{ilt}, u_{it}) = (L_{ih}, L_{il}, u_i). \tag{54}$$

Note that $l_{iht} = L_{iht}/n_i$, and $l_{ilt} = L_{ilt}/n_i$.

Lemma 5 *Given $(\theta_{at}, \theta_{mt}, L_{aht}, L_{alt}, L_{mht}, L_{mlt})$ at every $t \in [0, \infty)$, we obtain*

$$k_{it} = \Big[\frac{(1-\beta_{li}A_i^0)\beta_{ki}p_i a_{it}^{\beta_{ai}}\varepsilon_t^{\beta_{li}}(l_{ilt}+\alpha_i l_{iht})^{\beta_{li}}}{(r+\eta_k)p_k}\Big]^{\frac{1}{1-\beta_{ki}}} \tag{55}$$

$$a_{it} = \int_0^t e^{-\eta_a(t-\tau)}\lambda_i z_{i\tau}^{\kappa_i}(\varepsilon_\tau L_{ih\tau})^{1-\kappa_i}d\tau + e^{-\eta_a t}a_{i0}, \tag{56}$$

$$l_{iht} = L_{iht}/n_i, \quad \text{and} \quad l_{ilt} = L_{ilt}/n_i, \tag{57}$$

where

$$z_{it} = \varepsilon_t L_{iht}\Big[\frac{\lambda_i\kappa_i}{p_z}\int_t^\infty e^{-(r+\eta_a)(\tau-t)}(1-\beta_{li}A_i^0)\beta_{ai}p_i a_{i\tau}^{\beta_{ai}-1}k_{i\tau}^{\beta_{ki}}\varepsilon_\tau^{\beta_{li}}(l_{il\tau}+\alpha_i l_{ih\tau})^{\beta_{li}}d\tau\Big]^{\frac{1}{1-\kappa_i}}$$

Lemma 5 summarizes the dynamic path followed by entrepreneurs. Given $(\theta_{at}, \theta_{mt})$ at every $t \in [0, \infty)$, equations (24)-(26) jointly determine $(L_{aht}, L_{alt}, L_{mht}, L_{mlt}, u_{at}, u_{mt})$ and, together with Lemma 5, the unique path of the economy. Combining (20) and (27) yields

$$\gamma\varepsilon_t P^{-1} = q(\theta_{it})\frac{(1-\phi)(W_{ilt}-U_{it})}{\phi}, \tag{58}$$

which restores $(\theta_{at}, \theta_{mt})$ at every $t \in [0, \infty)$. Because $(W_{ilt}, U_{it}) \to (W_{il}, U_i)$, we can get the convergence point (θ_a, θ_m). As in the previous subsection, (58) raises the equilibrium restriction that $W_{ilt} - U_{it} > 0$ for all i and t. Equations (24)-(26) and (53)-(58) provide the full description of the transition path of the model.

Again, the complexity of the system prevents us from providing an analytical proof on the existence and uniqueness of the solution. Instead, we acknowledge that we obtain the same result when we repeat the experiment with 20 different sets of "initial guesses."

3 Numerical Analysis

This section presents a quantitative assessment of the underlying link between labor market institutions and economic growth, with a focus on two East Asian countries, Japan and South Korea. The Japanese episode is a comprehensive example of economic growth through structural transformation up to the starting point of 'lost decades'. Lack of data for the 1950s and 1960s precludes analysis of the initial periods of structural transformation independently of the 'postwar effect' of World War II (1939-45) and the Korean War (1950-53). Initially less confounded by the postwar effect, the structural transformation of South Korea accelerated in the early 1970s and continued until the Korean foreign currency crisis in 1997. This complementary analysis of the Japanese and South Korean episodes supports concrete conclusions about labor market institutions and economic growth in transitional economies.

In subsection 3.1, we calibrate the model by combining three Japanese data sources: the World Bank database; the Japanese national account; and the dataset employed by Hayashi and Prescott (2008). Lacking earlier data, we focus on the period beginning in 1960 and, to exclude the lost decades of Japan in the 1990s, ending in 1990. In subsection 3.2, we calibrate the model using three South Korean sources: the World Bank database; the Korean national account; and the estimation results presented by Chang, Nam, and Rhee (2004). We focus on the period starting in 1970, the implied starting point of structural transformation in South Korea, and, to avoid its foreign currency crisis, ending in 1995. [Table 1] and [Table 2] summarize our choices of parameter values associated with Japan's structural transformation, [Table 3] and [Table 4] our choices of parameter values associated with South Korea's structural transformation.

3.1 Japanese Structural Transformation

Exogenously Fed Parameters We set $r = 0.0095$ during the sample period to fix the annual interest rate at 3.8%, which is consistent with the estimate of the annual discount rate in Esteban-Pretel and Sawada (2009). But, the discount rate does not have a pronounced effect on the macro variables in our model. We normalize the price of non-agricultural products to be 1, and fix the relative price of agricultural products at 1.2, which is consistent with the average of the Engel coefficient from 1960-1990. The Engel coefficient, the ratio of food expenditures to total expenditures by households, is obtained by

$$\frac{p_a^{1-\sigma}}{p_a^{1-\sigma} + p_m^{1-\sigma}} \approx 0.37.$$

Japan's Engel coefficient was around 0.49 in 1960 and declined continuously to around 0.25 in 1990.[13] Because it assumes homothetic preferences, our model predicts a constant Engel coefficient, unless it additionally assumes the subsistence level, as in Matsuyama (1992). For the sake of simplicity, we alternatively take a simple average of the Engel coefficient during the target period.

The retirement rate is set to be 0.011, which results in roughly 90 percent of workers, who enter the labor market at age 25, retiring before age 70. The implied average 'market duration', the average elapsed time between labor market entry and exit, is thus less than than 25 years. Not being a lifecycle model with aging, our model considers a worker who moves into the non-labor force to be a retiree and a worker who moves from the non-labor force into the labor force to be a newly born worker. Average market duration is thus much shorter than average lifespan. Japan's population grew from 58.63 million in 1965 to 81.0 million in 1990.[14] The implied quarterly growth rate is $\ln(81.0/58.63)/100 \approx 0.003$, which determines the birth rate $\chi = \rho + 0.003 = 0.014$ during the sample period. The separation rate δ is set to 0.02 to obtain the average job duration of 12.5 years among non-retirees in the OECD data set. This choice is consistent with Esteban-Pretel and Fujimoto (2012). We posit

$$q(\theta_i) = 0.6 \times \theta_i^{-0.6} \quad \text{for each} \quad i \in \{a, m\}, \tag{59}$$

borrowing from Kano and Ohta (2002), who estimate the matching function of the Japanese labor market consistent with the plausible range of empirical elasticity of 0.5 to 0.7 proposed by Petrongolo and Pissarides (2001). We equalize the bargaining parameter to the elasticity of the matching technology by invoking Hosios's condition, that is, $\phi = 0.6$. These choices on bargaining parameter and matching function parameters are consistent with Esteban-Pretel and Fujimoto (2012). The growth rate of labor augmenting technology ψ is set at 0.007. The latter choice, which implies a balanced growth path of 0.028 annually, is based on the average annual growth rate (a weighted average of country growth rates) of GDP in all OECD countries for the period 1980-1990.[15]

The capital depreciation rate η_k, calculated as the ratio of depreciation to real capital stock each year, results in $\eta_k = 0.033$, following Jorgenson (1996).[16] Calculating the technology depreciation rate η_a based on the depreciation rate of service industry equipment results in $\eta_a = 0.041$, again following Jorgenson (1996). Due to the lack of sectoral data, we employ the aggregate economy level of the depreciation rate. The elasticity of substitution σ is set to 3.8, following Bernard, Eaton, Jensen, and Kortum (2003), Felbermayr, Prat, and Schmerer (2011), and Ishimaru, Oh, and Sim (2013). Values exogenously assigned are summarized in [Table 1].

[13] The Family Income and Expenditure Survey of the Ministry of Internal Affairs and Communications Statistics Bureau of Japan provides total consumption expenditure and food expenditure.

[14] These values are obtained from the Japanese Population Census, which can be downloaded from the Statistics Bureau of Japan.

[15] Their lost decades starting from 1990's preclude calibration based on the balanced growth path. Instead, we use the average growth rate of other OECD countries for the period 1980-1990. Table V.1. Growth Performance in OECD countries is from OECD Outlook 2000 V. Recent Growth Trends in OECD Countries.

[16] Calculating the average annual depreciation rate of agricultural (0.0971), construction (0.1722), mining and oil field (0.1650), metalworking (0.1225), and special industry machinery (0.1031) results in 0.132 annually, or 0.033 quarterly.

Calibrated Parameters For the remaining parameters, we choose a vector that matches the aggregate of the time series data for sectoral GDP growth per capita, sectoral labor share, sectoral capital share, sectoral wage growth, the net EXP/GDP, replacement ratio and average unemployment rate.

In [Figure 1], the dots represent actual data values, the smooth lines the time series predicted by the model. Overall, [Figure 1] suggests that our calibration strategy captures fairly well the trend in transition. Panels (a) and (b) present the time-series data and the model's prediction of sectoral GDP growth per capita from 1970 to 1990 (lacking data for earlier periods). Panels (c) and (d) report sectoral labor shares, panels (e) and (f) sectoral capital shares, from 1960-1990. Both labor and capital shares declined sharply in the agricultural, and rose rapidly in the non-agricultural, sector. In Panels (g)-(h), we exploit the evolution of sectoral wages from 1960-1990 to calibrate the labor productivity parameters $(\alpha_a, \alpha_m, \zeta)$. Based on historical wage data reported by Japan's Ministry of Health, Labor and Welfare,[17] from 1960-1990 workers' real wages grew by 29 and 152 percent in the agricultural and non-agricultural sectors, respectively. Panel (i) reports the unemployment rate in the actual data and the model. Without business cycle fluctuation, we set the target of the average unemployment rate at 2.0 percent. Panel (j) shows the Net Export/GDP ratio predicted by the model to be reconciled with the average Net Export/GDP ratio of 10.5 percent and its linear trend from 1960 to 1990. Panel (k) presents the population growth from the data and the model. Panel (l) reflects adjustment of the model's parameters to keep unemployment benefits at around 40 percent of the average wage over time. These choices are summarized in [Table 2].

[17] URL: www.mhlw.go.jp.

Table 1: Parameter Values: Exogenously Assigned (Japan)

Parameters	Description (Source/ Target)
$r = 0.0095$	discount rate (Esteban-Pretel and Sawada (2009))
$p_a/p_m = 1.2$	price of agriculture good (Engel coefficient)
$\rho = 0.011$	retirement rate (retirement age)
$\chi = 0.014$	birth rate (population growth rate)
$\delta = 0.02$	separation rate (Esteban-Pretel and Fujimoto (2012))
$q(\theta) = 0.6\theta^{-0.6}$	matching technology (Kano and Ohta (2002))
$\phi = 0.6$	bargaining parameter (Hosios (1990))
$\psi = 0.007$	growth rate of the labor augmenting technology (average growth rate of OECD countries)
$\eta_a = 0.041$	technology depreciation rate (Jorgenson (1996))
$\eta_k = 0.033$	capital depreciation rate (Jorgenson (1996))
$\sigma = 3.8$	elasticity of substitution (Ishimaru, Oh, and Sim (2013))

Table 2: Parameter Values: Endogenously Targeted (Japan)

Parameters	Description (Source/ Target)	
$\omega = 0.03$	sensitivity of inter-sectoral migration	
$\xi = 0.007$	arrival rate of revision shock	
$\alpha_a = 1.05$	agr labor Productivity	
$\alpha_m = 1.5$	non-Agr labor Productivity	
$\zeta = 0.6$	human capital accumulation	(the time series of
$\gamma = 0.12$	cost of vacancy	sectoral GDP growth,
$p_k = 0.3$	capital cost	sectoral labor share,
$p_z = 0.4$	technology investment cost	sectoral capital share,
$\beta_{ka} = 0.193$	agr capital share in production	sectoral wage growth,
$\beta_{km} = 0.23$	non-Agr capital share in production	net EXP/GDP ratio,
$\beta_{la} = 0.58$	agr labor share in production	replacement ratio, and
$\beta_{lm} = 0.5$	non-Agr labor share in production	unemployment rate)
$\lambda_a = 0.3$	efficiency of technology investment	
$\lambda_m = 0.5$	efficiency of technology investment	
$\kappa_a = 0.1$	elasticity of technology investment	
$\kappa_m = 0.4$	elasticity of technology investment	
$b = 1.3$	unemployment benefit	

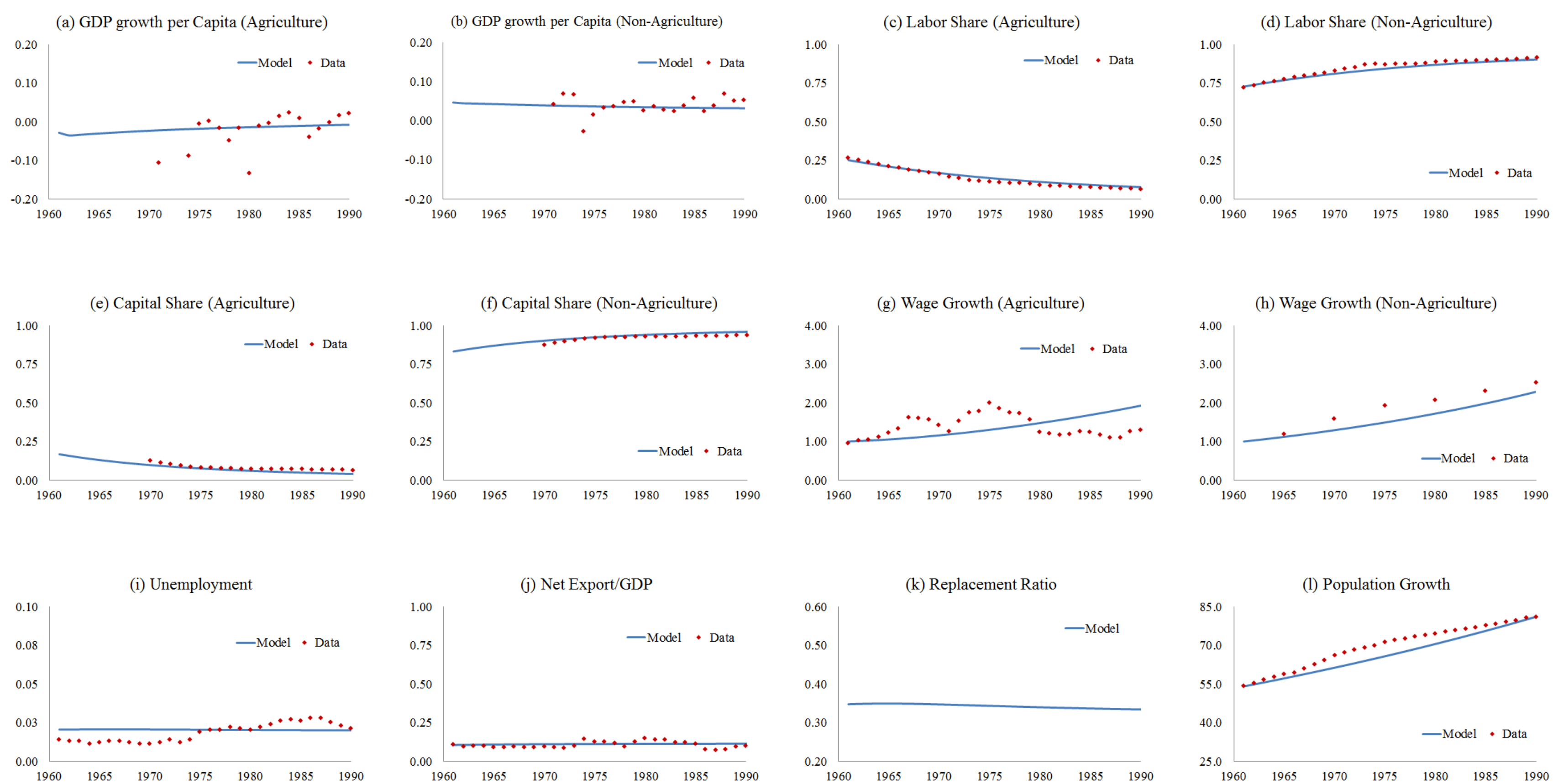

Figure 1: Calibration Results (Japan)

3.2 South Korean Structural Transformation

Exogenously Fed Parameters We set the real interest rate at 0.01 to match South Korea's annual average real interest rate of around 4.0 percent during 1970-1995 period.[18] As in the Japanese episode, we normalize the price of non-agricultural products to be 1 and fix the relative price of agricultural products at 1.2, consistent with the historical average Engel coefficient of 0.37 for South Korea for the 1970-1995 period. The share of food expenditure to total expenditure declined from 0.44 in 1970 to 0.26 in 1995. The retirement rate is set at 0.011 for consistency, and South Korea's quarterly growth rate was also 0.003, which implies the birth rate $\chi = \rho + 0.003 = 0.014$ during the sample period. According to World Bank data, South Korea's population grew from 31.92 million in 1970 to 45.09 million in 1995, which implies a quarterly growth rate of 0.003.

The separation rate δ is set to 0.027 based on the estimation results in Chang, Nam, and Rhee (2004), who show that the monthly separation rate of the aggregate economy declined from 0.012 in 1981 to 0.006 in 1994. Note that taking the average of these two values results in a quarterly separation rate of 0.027 and corresponding average job duration of 9.25 years. We also borrow from Chang, Nam, and Rhee (2004) the estimates of the matching technology in the South Korean labor market.

$$q(\theta_i) = 0.45 \times \theta_i^{-0.5} \quad \text{for each} \quad i \in \{a, m\} \tag{60}$$

Invoking Hosios's condition, we set $\phi = 0.5$. We also set the growth rate of the labor augmenting technology ψ at 0.01, which is equivalent to an annual GDP growth rate of 4.0 percent on the balanced growth path based on South Korea's average annual GDP per capita growth rate from 2000 to 2010.[19] We use the same values as in the Japanese episode for the remaining parameters (η_a, η_k, σ).

Calibrated Parameters We choose a vector of the remaining parameters to match the aggregate of the time series data for sectoral GDP growth per capita, sectoral labor share, the EXP/GDP ratio, capital investment growth, sectoral value-added share, and the net EXP/GDP and replacement ratios and average unemployment rate. Given the similarity between the Japanese and Korean economies and lack of sectoral capital share data, we keep the same parameter values for $(p_k, p_z, \beta_{ka}, \beta_{km}, \beta_{la}, \beta_{lm})$ as in the Japanese episode. Most of the panels in [Figure 2] report the Korean equivalents of the Japanese data. The exceptions are sectoral capital share and wage growth. The latter time series for the sample period being unavailable in the South Korean dataset, we substitute the time-series data for the EXP/GDP ratio in Panel (e), the aggregate capital investment growth rate in Panel (f), and the sectoral value-added shares in Panels (e) and (f). The actual data show the labor and value-added shares to have declined substantially in the agricultural, and risen sharply in the non-agricultural, sector. Overall, [Figure 2] suggests that our calibration strategy captures fairly well the behavior of the target time series data for South Korea for 1970-1995 period.

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[18] Chang, Nam, and Rhee (2004) use the annual real interest rate of 4.17 percent during the 1975-1994 period.

[19] Because the Japanese economy is suffering still from the ongoing 'lost decades', whereas the South Korean economy was considered recovered from the foreign currency crisis in the early 2000s, we adopt different calibration strategies for their potential growth rates on the balanced growth path.

Table 3: Parameter Values: Exogenously Assigned (South Korea)

Parameters	Description (Source/ Target)
$r = 0.01$	discount rate (Chang, Nam, and Rhee (2004))
$p_a/p_m = 1.2$	price of agriculture good (Engel coefficient)
$\rho = 0.011$	retirement rate (retirement age)
$\chi = 0.014$	birth rate (population growth rate)
$\delta = 0.027$	separation rate (Chang, Nam, and Rhee (2004))
$q(\theta) = 0.45\theta^{-0.5}$	matching technology (Chang, Nam, and Rhee (2004))
$\phi = 0.5$	bargaining parameter (Hosios (1990))
$\psi = 0.01$	growth rate of the labor augmenting technology
	(the average growth rate of South Korea from 2000-2010)
$\eta_a = 0.041$	technology depreciation rate (Jorgenson (1996))
$\eta_k = 0.033$	capital depreciation rate (Jorgenson (1996))
$\sigma = 3.8$	elasticity of substitution (Ishimaru, Oh, and Sim (2013))

Table 4: Parameter Values: Endogenously Targeted (South Korea)

Parameters	Description (Source/ Target)	
$\omega = 0.035$	sensitivity of inter-sectoral migration	
$\xi = 0.006$	arrival rate of revision shock	
$\alpha_a = 1.1$	Agr labor Productivity	
$\alpha_m = 1.55$	Non-Agr labor Productivity	
$\zeta = 0.7$	human capital accumulation	(the time series of
$\gamma = 0.213$	cost of vacancy	sectoral GDP growth,
$p_k = 0.3$	capital cost	sectoral labor share,
$p_z = 0.4$	technology investment cost	sectoral value-added share,
$\beta_{ka} = 0.193$	Agr capital share in production	aggregate capital growth,
$\beta_{km} = 0.23$	Non-Agr capital share in production	EXP/GDP ratio,
$\beta_{la} = 0.58$	Agr labor share in production	net EXP/GDP ratio,
$\beta_{lm} = 0.5$	Non-Agr labor share in production	replacement ratio, and
$\lambda_a = 0.2$	efficiency of technology investment	unemployment rate)
$\lambda_m = 0.5$	efficiency of technology investment	
$\kappa_a = 0.1$	elasticity of tech. investment	
$\kappa_m = 0.45$	elasticity of tech. investment	
$b = 1.15$	unemployment benefit	

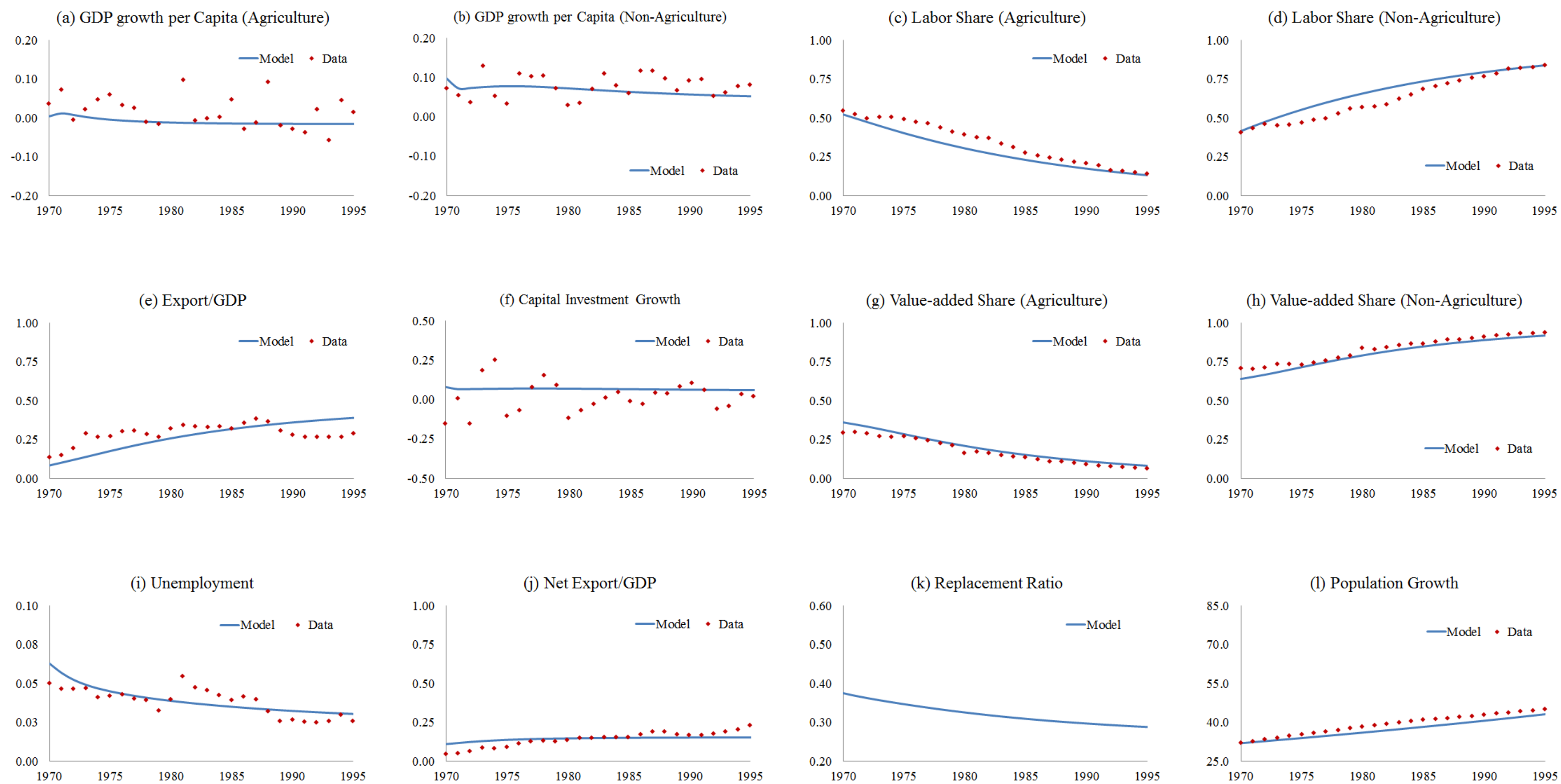

Figure 2: Calibration Results (South Korea)

4 Counterfactual Experiments

Using the calibrated model, we conduct counterfactual experiments by substituting different arrival rates of separation shock and matching technologies. The baseline simulations, represented by the solid lines, are the results with the calibrated parameters. In the first counterfactual experiment, represented by the dotted lines (Counterfactual Experiment 1, or CE-1), setting the separation rate at 0.239 and retirement rate at 0.011 results in a one-year job tenure. The second experiment, represented by the dashed lines (Counterfactual Experiment 2, or CE-2), posits the counterfactual question: "If the Japanese and South Korean labor markets had transplanted from the flexible U.S. labor market the efficient matching technology and high separation rate, what would have happened in 1990?"

[Figure 3] shows how varying job duration affects human capital accumulation and labor productivity, as calculated in each counterfactual experiment. Long job duration enables each entrepreneur to maintain high employment at a lower cost, and improves the average productivity of employed workers through learning-by-doing on the job. Panels (a)-(d) show the fraction of skilled workers to trend downward in the agricultural and upward in the non-agricultural sectors, and to be lower in counterfactual experiments involving a high separation rate. Panels (e)-(h) show the fraction of unskilled workers to exhibit moderate changes over time, Panels (i)-(l) labor productivity to be substantially higher in the baseline simulation.

[Figure 4] shows the transitional paths of technology stock, capital stock, and total production, as determined in each simulation. Panels (a)-(h) show agricultural investments in technology and capital to be slightly lower, and non-agricultural investments to be substantially lower, in the counterfactual experiments than in the baseline simulation. That technology and capital accumulation are more pronounced in the non-agricultural than in the agricultural sector is important. Taken together with the sectoral human capital accumulation patterns depicted in [Figure 3], [Figure 4] shows long job tenure and enhanced labor productivity to stimulate technology and capital investment more greatly in the non-agricultural sector. Consistent with these results, Panels (g)-(h) show aggregate production to be higher in the baseline simulation than in the counterfactual experiments. GDP growth in the non-agricultural sector, in which human capital accumulation significantly improves labor productivity and encourages investment, is especially accelerated by long job duration.

[Table 5] summarizes the main results of our numerical experiments in terms of key ratios of counterfactual simulation results to baseline simulation results in 1990. Because structural transformation was still ongoing in South Korea in 1990, these ratios are relatively underestimated in the South Korean experiments. Panel (a) presents the results from the first counterfactual experiment (CE-1), in which we set the separation rate at 0.239 and retirement rate at 0.011 to make job tenure one year. This result shows that if the job duration of Japanese and South Korean workers had been one year, the non-agricultural employment share in 1990 would have accounted for about 80 percent, and non-agricultural GDP per capita in 1990 for about 70 percent, of their actual values. This suggests that long job duration is a crucial determinant of structural transformation that enables the economy to achieve rapid growth by driving resource reallocation from the agricultural to the non-agricultural sector.

Panel (b) depicts the second counterfactual simulation model (CE-2), which shows that, whereas agricultural GDP per capita would have not been affected, non-agricultural GDP per capita would have been lower by 5-8 percent in both countries. At the level of the aggregate economy, the difference between the baseline simulation and CE-2 implies that lower job duration would have reduced total GDP per capita by 7 and 5 percent in Japan and South Korea, respectively. The counterfactual analysis of technology stock, capital stock, and human capital accumulation provides an explanation for the gap in GDP per capita across different labor market institutions. We show technology and capital stocks in the non-agricultural sector to be significantly lower in CE-2 than in the baseline simulations. In CE-2, the sectoral accumulations of technology and capital in the non-agricultural sector are 90-93 percent and 92-95 percent of the baseline simulation result in Japan and South Korea, respectively, and skilled labor in the non-agricultural sector declines by 11 percent in Japan and 9 percent in Korea, whereas unskilled labor in both sectors increases significantly in both countries. This indicates that high separation rates drive lower human capital accumulation.

Panel (c) illustrates the effect of only making job duration shorter, as in the US labor markets, without transplanting the matching technology. These counterfactual simulation results show GDP per capita to be lower than in the baseline simulation or CE-2, which indicates that a high exogenous separation rate would have had a greater impact, some part of which, however, would have been absorbed by the efficient matching technology in CE-2.

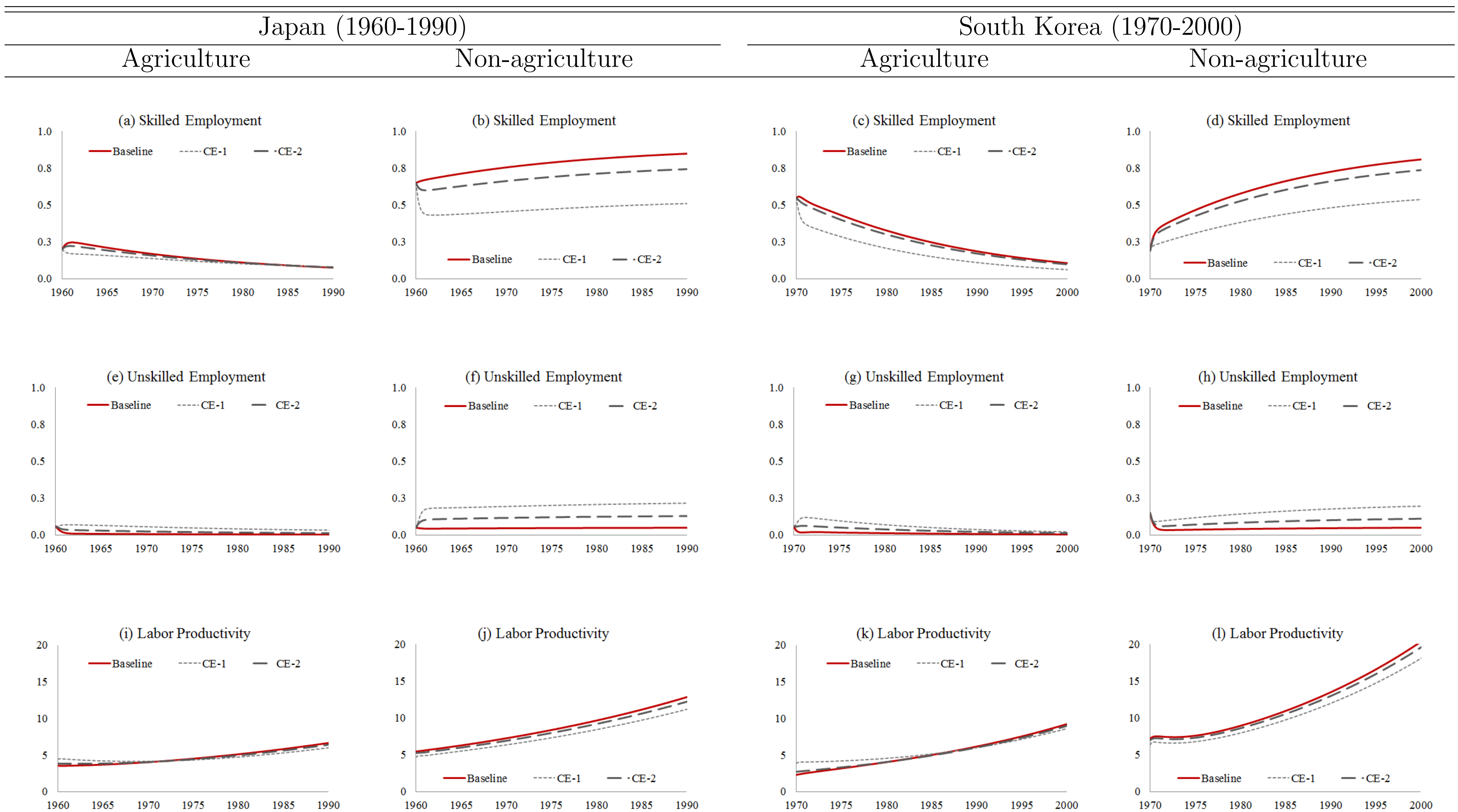

Figure 3: Counterfactual Results: Labor Market Institutions and Labor Productivity

Japan (1960-1990)		South Korea (1970-2000)	
Agriculture	Non-agriculture	Agriculture	Non-agriculture

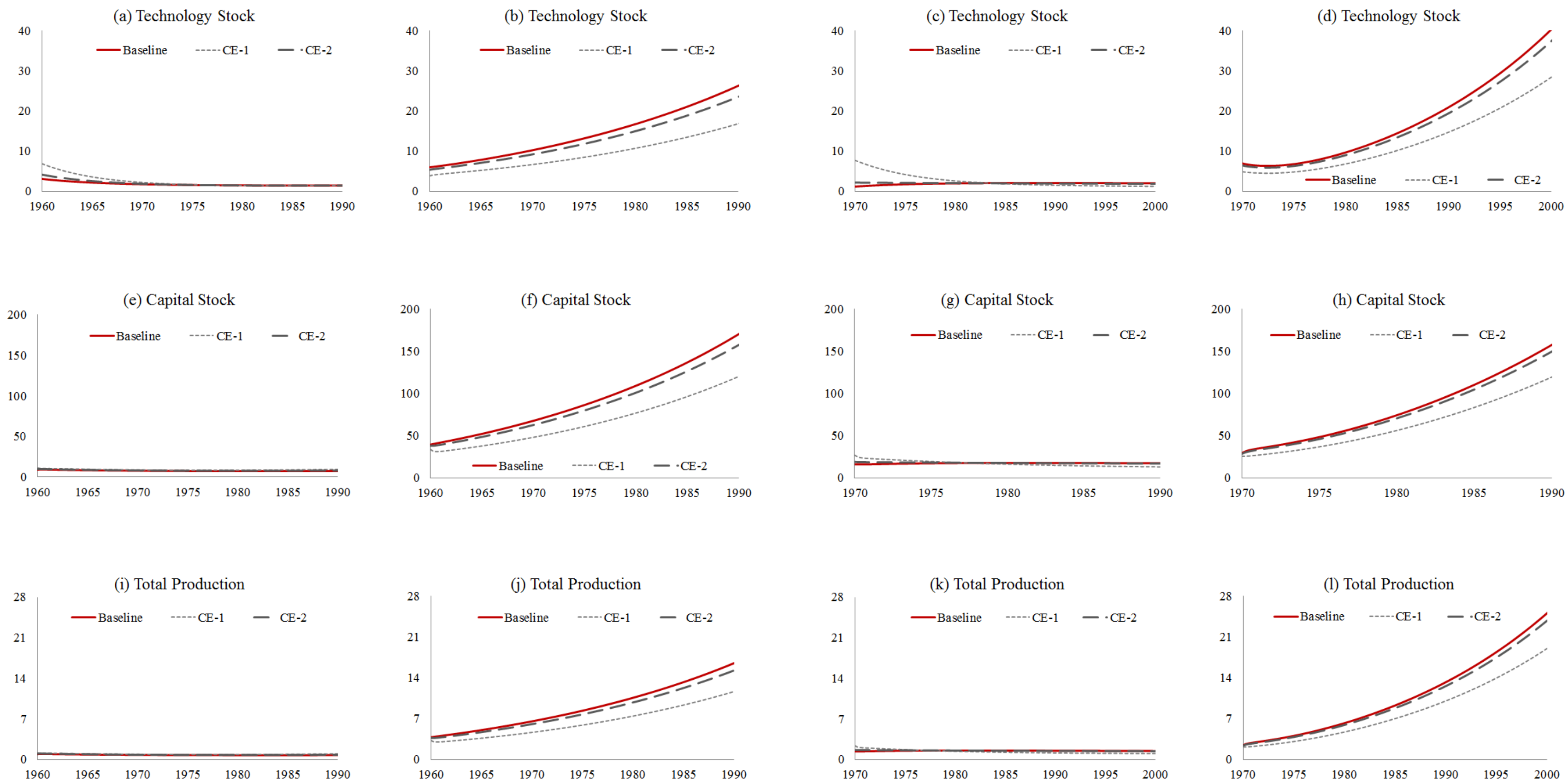

Figure 4: Counterfactual Results: Labor Market Institutions and Economic Growth

Table 5: Counterfactual Experiments

	Japan (1990)			South Korea (1990)		
	Agriculture	Non-Agriculture	Aggregate	Agriculture	Non-Agriculture	Aggregate
Panel A. Experiment with $\delta = 0.239$ to fix the job tenure at 1 year						
GDP per capita	1.29	0.71	0.74	0.75	0.76	0.76
Technology Stock	1.02	0.64	0.66	0.74	0.70	0.71
Capital Stock	1.29	0.71	0.73	0.75	0.76	0.76
Employment	1.43	0.81	0.86	0.76	0.85	0.84
Skilled Employment	1.06	0.60	0.64	0.59	0.66	0.65
Unskilled Employment	10.34	4.39	4.75	5.23	3.81	4.00
Panel B. Experiment with $\delta = 0.089$ and $q(\theta) = 1.35\theta^{-0.72}$						
GDP per capita	1.09	0.92	0.93	0.98	0.95	0.95
Technology Stock	1.01	0.90	0.90	0.94	0.93	0.93
Capital Stock	1.09	0.92	0.93	0.98	0.95	0.95
Employment	1.13	0.97	0.98	1.00	0.99	0.99
Skilled Employment	1.02	0.88	0.89	0.92	0.91	0.91
Unskilled Employment	3.84	2.61	2.69	3.04	2.17	2.29
Panel C. Experiment with $\delta = 0.089$						
GDP per capita	1.08	0.90	0.91	0.88	0.93	0.92
Technology Stock	0.99	0.87	0.88	0.86	0.91	0.90
Capital Stock	1.08	0.90	0.91	0.88	0.93	0.92
Employment	1.12	0.94	0.96	0.89	0.97	0.95
Skilled Employment	1.01	0.85	0.86	0.82	0.89	0.88
Unskilled Employment	3.80	2.54	2.62	2.70	2.12	2.20

5 Conclusion

We incorporate labor market friction and learning-by-doing into a transitional two-sector growth framework to create a new endogenous growth model for exploring the underlying link between labor market institutions and economic growth. As workers stay in their jobs longer, they become more productive through learning-by-doing, which concomitantly lowers the labor cost to entrepreneurs. Enhanced labor productivity stimulates physical capital investment by forward-looking entrepreneurs. When the economy specializes in a sector where learning-by-doing does not improve labor productivity significantly, such as the agricultural sector, economic growth is more moderate. When the economy shifts towards the industrialized sector, on the other hand, in which the effect of learning-by-doing is significant, a stable labor market with a long job duration stimulates and accelerates economic growth by encouraging investment as well as improving labor productivity.

In the numerical analysis, we apply our model to the East Asian episode of structural transformation. Those countries' lifetime employment systems, borne of the Confucian tradition that ethically discourages job turnover by employees and imposes on employers the duty of continuing employment, contributes to the formation of Ricardian comparative advantage in the non-agricultural industrialized sector. The counterfactual experiment finds that had the average job duration of Japanese and South Korean workers been one year, the non-agricultural employment share of each country would have accounted for about 80 percent, and non-agricultural GDP per capita for about 70 percent, of their actual values in 1990, which suggests sluggish structural transformation. Had the Japanese and Korean labor markets transplanted and maintained the efficient matching technology and high separation rate of the flexible U.S. labor market, non-agricultural GDP per capita in 1990 would have declined by 8 percent in Japan and 5 percent in South Korea, and aggregate GDP per capita by 7 percent and 5 percent, respectively. Lengthy job tenure was apparently one of the key ingredients in the structural transformation towards the non-agricultural sector.

Overall, this paper studies the underlying link between labor market institutions and the economic growth in transitional economies by highlighting matches between particular labor market institutions and sectoral characteristics. We hope that it delivers useful implications for many developing countries that are currently experiencing or expecting structural transformation towards the industrial sector. One may ask if our model can account for the recent growth slowdown or stagnation in NICs and Japan. This paper focuses only on structural transformation from the agricultural to non-agricultural (especially manufacturing) sectors, with an emphasis on the 'capitalization effect'. In contrast, the emergence after the 1990s of new service sectors, such as information technology and the health and financial industries, may stimulate 'creative destruction'. Potential requirements for different processes of human capital accumulation and/or different types of human capital, being beyond the scope of this paper, are left to future research.

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A Numerical Algorithm

A.1 On the Balanced Growth Path

In this appendix, we describe the computational algorithm for solving the balanced growth path. To start, guess $\{\theta_a, \theta_b\}$.

(a) Given (θ_a, θ_m), we obtain $(U_{mt} - U_{at})/\varepsilon_t$ and (ω_a, ω_m) using (4) and (36).

(b) Let

$$\begin{aligned} LHS_i &= [\gamma(r+\delta+\rho-\psi)/(\beta_{li}q(\theta_i)) + A^1_{it}/(\beta_{li}\varepsilon_t)]^{1-\beta_{ki}-\beta_{ai}\kappa_i}, \quad \text{and} \\ RHS_i &= \left[\frac{\lambda_i m_i}{\chi-\rho+\psi+\eta_a}\right]^{\beta_{ai}(1-\kappa_i)} \left[\frac{\lambda_i \kappa_i \beta_{ai}}{p_z(r+\eta_a)}\right]^{\beta_{ai}\kappa_i} \left[\frac{\beta_{ki}}{p_k(r+\eta_k)}\right]^{\beta_{ki}} (1-\beta_{li}A^0_i)p_i. \end{aligned}$$

If $\sum_i (RHS_i - LHS_i)^2$ is less than the preassigned tolerance level, go to step (c). Otherwise, using the Nelder and Meade method, update (θ_a, θ_m) and go back to step (a).

(c) Given (θ_a, θ_m), solve for variables $\{L_i, u_i, w_i, k_i, x_i, a_i, z_i, v_i, Y_i\}_{i\in\{a,m\}}$.

A.2 On the Transition Path

In this subsection, we present the solution algorithm for the transitional path to the new balanced growth path. First, we analyze the transition path from an initial point to the autarky balanced growth path. Assume that the economy converges to the new balanced growth path within T. We already know both endpoints.

(a) Pick up a sufficiently large amount of time for T and construct the set of evenly spaced grid points $\{t_0, t_1, \cdots, t_n\} \subset [0, T]$.

(b) Guess the entire transition path of $(L_{at}, L_{mt}, u_{at}, u_{mt}, W_{at}, W_{mt}, U_{at}, U_{mt}, p_{at}, p_{mt})$.

(c) Take all the other series as given and calculate the new series of $(\hat{L}_{at}, \hat{L}_{mt}, \hat{u}_{at}, \hat{u}_{mt})$ by forward shooting. Then, update $L_{at} = (1-a)L_{at} + a\hat{L}_{at}$ for a sufficiently small but positive a. Using the same weight a, update $(\hat{L}_{mt}, \hat{u}_{at}, \hat{u}_{mt})$.

(d) Take all the other series as given and iterate the new series of $(\hat{W}_{at}, \hat{W}_{mt}, \hat{U}_{at}, \hat{U}_{mt})$ by the backward shooting and weighted updating procedure as in step (c).

(e) Take all the other series as given and iterate the prices $(\hat{p}_{at}, \hat{p}_{mt})$ using the market clearing condition and weighted updating procedure. In case of the open economy, skip this step.

(f) Iterate step (c), (d), and (e), until all the series converge to the below of a certain tolerance level. If the differentials between the initial value and the updated values are small enough, move onto step (g).

(g) Prolong the time interval into $[0, \tilde{T}]$, where $\tilde{T} > T$. Repeat step (b)-(f). If the (point-wise) maximum difference between the two sets of the converged series in $[0, T]$ is below a certain tolerance level, stop here. Otherwise, update $T = \tilde{T}$, enlarge $\tilde{T}$ and repeat step (a)-(g).

B Mathematical Appendix

Proof of Lemma 1 The entrepreneur chooses $(x_{i\tau}, z_{i\tau}, v_{i\tau})$ at every $\tau \in [t, \infty)$ to maximize

$$E_{it}(\bar{a}_i, \bar{k}_i, \bar{l}_{ih}, \bar{l}_{il}) = \max_{z_{i\tau}, x_{i\tau}, v_{i\tau} \geq 0} \int_t^\infty e^{-r(\tau - t)} \pi_{i\tau} P^{-1} d\tau \tag{B1}$$

subject to

$$\begin{aligned}
\dot{a}_{i\tau} &= -\eta_a a_{i\tau} + \lambda_i z_{i\tau}^{\kappa_i} (\varepsilon_\tau L_{ih\tau})^{1-\kappa_i} && \text{(B2)} \\
\dot{k}_{i\tau} &= -\eta_k k_{i\tau} + x_{i\tau} && \text{(B3)} \\
\dot{l}_{ih\tau} &= -(\delta + \rho) l_{ih\tau} + \zeta l_{il\tau} && \text{(B4)} \\
\dot{l}_{il\tau} &= -(\delta + \rho + \zeta) l_{il\tau} + q(\theta_{i\tau}) v_{i\tau} && \text{(B5)} \\
a_{it} &= \bar{a}_i, \quad k_{it} = \bar{k}_i, \quad l_{iht} = \bar{l}_{ih}, \quad \text{and} \quad l_{ilt} = \bar{l}_{il} && \text{(B6)}
\end{aligned}$$

First, we ignore the non-negative restriction in the domain and solve for the optimal control problem. Then, we check if the optimal decision is binding. The Hamiltonian for the above problem is

$$\begin{aligned}
\mathcal{H} \;=\; & e^{-r(\tau-t)} [p_i a_{i\tau}^{\beta_{ai}} k_{i\tau}^{\beta_{ki}} \varepsilon_\tau^{\beta_{li}} (l_{il\tau} + \alpha_i l_{ih\tau})^{\beta_{li}} - \sum_{j=h,l} w_{ij\tau} l_{ij\tau} - p_k x_{i\tau} - \gamma \varepsilon_\tau v_{i\tau} - p_z z_{i\tau}] P^{-1} \\
& - \mu_a [\eta_a a_{i\tau} - \lambda_i z_{i\tau}^{\kappa_i} (\varepsilon_\tau L_{ih\tau})^{1-\kappa_i}] - \mu_k [\eta_k k_{i\tau} - x_{i\tau}] \\
& - \mu_h [(\delta + \rho) l_{ih\tau} - \zeta l_{il\tau}] - \mu_l [(\delta + \rho + \zeta) l_{il\tau} - q(\theta_{i\tau}) v_{i\tau}].
\end{aligned}$$

The maximum principle implies that

$$\begin{aligned}
z_{i\tau} &: \; e^{-r(\tau-t)} P^{-1} p_z z_{i\tau}^{1-\kappa_i} = \mu_a \kappa_i \lambda_i (\varepsilon_\tau L_{ih\tau})^{1-\kappa_i} && \text{(B7)} \\
x_{i\tau} &: \; e^{-r(\tau-t)} P^{-1} p_k = \mu_k && \text{(B8)} \\
v_{i\tau} &: \; e^{-r(\tau-t)} P^{-1} \varepsilon_\tau \gamma = \mu_l q(\theta_{i\tau}) && \text{(B9)} \\
a_{i\tau} &: \; \dot{\mu}_a = -e^{-r(\tau-t)} P^{-1} \frac{\partial \pi_{i\tau}}{\partial a_{i\tau}} + \mu_a \eta_a && \text{(B10)} \\
k_{i\tau} &: \; \dot{\mu}_k = -e^{-r(\tau-t)} P^{-1} \frac{\partial \pi_{i\tau}}{\partial k_{i\tau}} + \mu_k \eta_k && \text{(B11)} \\
l_{ih\tau} &: \; \dot{\mu}_h = -e^{-r(\tau-t)} P^{-1} \frac{\partial \pi_{i\tau}}{\partial l_{ih\tau}} + \mu_h (\delta + \rho) && \text{(B12)} \\
l_{il\tau} &: \; \dot{\mu}_l = -e^{-r(\tau-t)} P^{-1} \frac{\partial \pi_{i\tau}}{\partial l_{il\tau}} - \mu_h \zeta + \mu_l (\delta + \rho + \zeta) && \text{(B13)}
\end{aligned}$$

From (B10),

$$\begin{aligned}
& e^{-\eta_a(\tau-t)} \dot{\mu}_a - \eta_a e^{-\eta_a(\tau-t)} \mu_a = -e^{-(r+\eta_a)(\tau-t)} P^{-1} \frac{\partial \pi_{i\tau}}{\partial a_{i\tau}} \\
\iff \quad & \mu_a = e^{\eta_a(\tau-t)} \int_\tau^\infty e^{-(r+\eta_a)(\tau'-t)} P^{-1} \frac{\partial \pi_{i\tau'}}{\partial a_{i\tau'}} d\tau' + C_a e^{\eta_a(\tau-t)}
\end{aligned}$$

Since the shadow price μ_a cannot diverge as $\tau \to \infty$, $C_a = 0$. Thus, we get

$$\mu_a = e^{\eta_a(\tau-t)} \int_\tau^\infty e^{-(r+\eta_a)(\tau'-t)} P^{-1} \frac{\partial \pi_{i\tau'}}{\partial a_{i\tau'}} d\tau' \tag{B14}$$

Then, plugging (B14) into (B7) yields

$$z_{it} = \varepsilon_t L_{iht}\Big[\frac{\lambda_i \kappa_i}{p_z}\int_t^\infty e^{-(r+\eta_a)(\tau-t)}\frac{\partial \pi_{i\tau}}{\partial a_{i\tau}}d\tau\Big]^{\frac{1}{1-\kappa_i}} \tag{B15}$$

Along the similar reasoning, by combining (B8) and (B11), we get

$$p_k = \int_t^\infty e^{-(r+\eta_k)(\tau-t)}\frac{\partial \pi_{i\tau}}{\partial k_{i\tau}}d\tau. \tag{B16}$$

From (B12) and (B13), we get

$$\mu_h = e^{(\delta+\rho)(\tau-t)}\int_\tau^\infty e^{-(r+\delta+\rho)(\tau'-t)}P^{-1}\frac{\partial \pi_{i\tau'}}{\partial l_{ih\tau'}}d\tau', \quad \text{and} \tag{B17}$$

$$\begin{aligned}\mu_l &= e^{(\delta+\rho+\zeta)(\tau-t)}\int_\tau^\infty e^{-(r+\delta+\rho+\zeta)(\tau'-t)}P^{-1}\Big[\frac{\partial \pi_{i\tau'}}{\partial l_{il\tau'}} - \frac{\partial \pi_{i\tau'}}{\partial l_{ih\tau'}}\Big]d\tau' \\ &\quad + e^{(\delta+\rho)(\tau-t)}\int_\tau^\infty e^{-(r+\delta+\rho)(\tau'-t)}P^{-1}\frac{\partial \pi_{i\tau'}}{\partial l_{ih\tau'}}d\tau'\end{aligned} \tag{B18}$$

Plugging (B18) into (B9) yields

$$\frac{\varepsilon_t\gamma}{P} = q(\theta_{it})\int_t^\infty e^{-(r+\delta+\rho)(\tau-t)}P^{-1}\Big[e^{-\zeta(\tau-t)}\frac{\partial \pi_{i\tau}}{\partial l_{il\tau}} + (1-e^{-\zeta(\tau-t)})\frac{\partial \pi_{i\tau}}{\partial l_{ih\tau}}\Big]d\tau \tag{B19}$$

Finally, since

$$\frac{\partial E_{it}}{\partial l_{ilt}} = \int_t^\infty e^{-(r+\delta+\rho)(\tau-t)}P^{-1}\Big[e^{-\zeta(\tau-t)}\frac{\partial \pi_{i\tau}}{\partial l_{il\tau}} + (1-e^{-\zeta(\tau-t)})\frac{\partial \pi_{i\tau}}{\partial l_{ih\tau}}\Big]d\tau, \tag{B20}$$

connecting (B19) and (B20) results in

$$P^{-1}\varepsilon_t\gamma = q(\theta_{it})\frac{\partial E_{it}}{\partial l_{ilt}} \tag{B21}$$

Proof of Lemma 2 Let $E^h_{iht} := \frac{\partial E_{iht}}{\partial l_{iht}}$ and $E^l_{ilt} := \frac{\partial E_{ilt}}{\partial l_{ilt}}$. From the bargaining rule proposed by Stole and Zwiebel (1996),

$$(1-\phi)(W_{ijt} - U_{it}) = \phi E^j_{ijt} \quad \text{and} \quad (1-\phi)(\dot{W}_{ijt} - \dot{U}_{it}) = \phi \dot{E}^j_{ijt} \tag{B22}$$

By equation (B21) and (B22), we get as follows.

$$(1-\phi)(w_{iht} - b\varepsilon_t - \xi\Delta_{it}P) = \phi(\frac{\partial \pi_{it}}{\partial l_{iht}} + \varepsilon_t\gamma\theta_{it}) \tag{B23}$$

$$(1-\phi)(w_{ilt} - b\varepsilon_t - \xi\Delta_{it}P) = \phi(\frac{\partial \pi_{it}}{\partial l_{ilt}} + \varepsilon_t\gamma\theta_{it}) \tag{B24}$$

Rewriting this yields

$$w_{iht} + \phi\sum_{j=l,h}\frac{\partial w_{ijt}}{\partial l_{iht}}l_{ijt} = \phi p_i\frac{\partial y_{it}}{\partial l_{iht}} + (1-\phi)(b\varepsilon_t + \xi\Delta_{-it}P^{-1}) + \phi\varepsilon_t\gamma\theta_{it} \tag{B25}$$

$$w_{ilt} + \phi \sum_{j=l,h} \frac{\partial w_{ijt}}{\partial l_{ilt}} l_{ijt} = \phi p_i \frac{\partial y_{it}}{\partial l_{ilt}} + (1-\phi)(b\varepsilon_t + \xi \Delta_{-it} P^{-1}) + \phi \varepsilon_t \gamma \theta_{it} \tag{B26}$$

The solution of the above differential equation is given by

$$w_{iht} = \alpha_i A_i^0 p_i \frac{\partial y_{it}}{\partial l_{ilt}} + A_{it}^1, \quad \text{and} \quad w_{ilt} = A_i^0 p_i \frac{\partial y_{it}}{\partial l_{ilt}} + A_{it}^1 \tag{B27}$$

where

$$A_i^0 = \frac{\phi}{1-\phi(1-\beta_{li})} \quad \text{and} \quad A_{it}^1 = (1-\phi)(b\varepsilon_t + \xi \Delta_{-it} P) + \varepsilon_t \gamma \phi \theta_{it} \tag{B28}$$

Proof of Lemma 3 Rewriting (39) and (40) in matrix notation yields

$$\begin{pmatrix} \chi+\delta & 0 & -f(\theta_a) & 0 \\ 0 & \chi+\delta & 0 & -f(\theta_m) \\ -\delta & 0 & \chi + f(\theta_a) + \xi\omega_m & -\xi\omega_a \\ 0 & -\delta & -\xi\omega_m & \chi + f(\theta_m) + \xi\omega_a \end{pmatrix} \begin{pmatrix} L_{ah} + L_{al} \\ L_{mh} + L_{ml} \\ u_a \\ u_m \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \chi\omega_a \\ \chi\omega_m \end{pmatrix}$$

The matrix on the left-hand side is non-singular. Multiplying the inverse matrix of it on both side yields $(L_{ah} + L_{al}, L_{mh} + L_{ml}, u_a, u_m)$. Since $(\chi+\delta)L_{ih} = \zeta L_{il}$ for each $i \in \{a, m\}$ on steady states, we get $(L_{ah}, L_{al}, L_{mh}, L_{ml}, u_a, u_m)$.

Proof of Lemma 4 Since

$$\frac{\partial \pi_{it}}{\partial a_{it}} = \beta_{ai} p_i (1-\beta_{li} A_i^0) a_{it}^{\beta_{ai}} k_{it}^{\beta_{ki}} [\varepsilon_{it}(l_{ilt} + \alpha_i l_{iht})]^{\beta_{li}} \frac{1}{a_{it}}, \tag{B29}$$

we get

$$\begin{aligned} z_{it} &= \varepsilon_t L_{iht} \Big[\frac{\lambda_i \kappa_i P}{p_z} \int_t^\infty e^{-(r+\eta_a)(\tau-t)} P^{-1} \frac{\partial \pi_{i\tau}}{\partial a_{i\tau}} d\tau \Big]^{\frac{1}{1-\kappa_i}} \\ &= \varepsilon_t L_{iht} \Big[\frac{\lambda_i \kappa_i \beta_{ai} p_i (1-\beta_{li} A_i^0)}{p_z (r+\eta_a) \beta_{li} a_{it}} \frac{\partial y_{it}}{\partial l_{ilt}} (l_{ilt} + \alpha_i l_{iht}) \Big]^{\frac{1}{1-\kappa_i}}, \end{aligned}$$

and

$$\begin{aligned} a_{it} &= \frac{\lambda_i \varepsilon_t L_{iht}}{\chi - \rho + \psi + \eta_a} \Big[\frac{\lambda_i \kappa_i \beta_{ai} p_i (1-\beta_{li} A_i^0)}{p_z (r+\eta_a) \beta_{li} a_{it}} \frac{\partial y_{it}}{\partial l_{ilt}} (l_{ilt} + \alpha_i l_{iht}) \Big]^{\frac{\kappa_i}{1-\kappa_i}} \\ &= \Big[\frac{\lambda_i \varepsilon_t L_{iht}}{\chi - \rho + \psi + \eta_a} \Big]^{1-\kappa_i} \Big[\frac{\lambda_i \kappa_i \beta_{ai} p_i (1-\beta_{li} A_i^0)}{\beta_{li} p_z (r+\eta_a)} \frac{\partial y_{it}}{\partial l_{ilt}} (l_{ilt} + \alpha_i l_{iht}) \Big]^{\kappa_i}. \end{aligned} \tag{B30}$$

From (19),

$$p_k = \int_t^\infty e^{-(r+\eta_k)(\tau-t)} \frac{\partial \pi_{i\tau}}{\partial k_{i\tau}} d\tau = \int_t^{t+dt} e^{-(r+\eta_k)(\tau-t)} \frac{\partial \pi_{i\tau}}{\partial k_{i\tau}} d\tau + e^{-(r+\eta_k)dt} p_k$$

Then, reordering and dividing by dt yields

$$\frac{1-e^{-(r+\eta_k)dt}}{dt} p_k = \frac{1}{dt} \int_t^{t+dt} e^{-(r+\eta_k)(\tau-t)} \frac{\partial \pi_{i\tau}}{\partial k_{i\tau}} d\tau$$

Finally, sending $dt \to 0$ and reordering results in

$$k_{it} = \Big[\frac{(1-\beta_{li}A_i^0)\beta_{ki}p_i a_{it}^{\beta_{ai}}\varepsilon_t^{\beta_{li}}(l_{ilt}+\alpha_i l_{iht})^{\beta_{li}}}{(r+\eta_k)p_k}\Big]^{\frac{1}{1-\beta_{ki}}} \quad \text{and} \tag{B31}$$

$$x_{it} = (\chi-\rho+\psi+\eta_k)\Big[\frac{(1-\beta_{li}A_i^0)\beta_{ki}p_i a_{it}^{\beta_{ai}}\varepsilon_t^{\beta_{li}}(l_{ilt}+\alpha_i l_{iht})^{\beta_{li}}}{(r+\eta_k)p_k}\Big]^{\frac{1}{1-\beta_{ki}}}. \tag{B32}$$

Plugging (B30) and (B31) into the expression of $\frac{\partial y_{it}}{\partial l_{ilt}}$ and reordering yields

$$\frac{\partial y_{it}}{\partial l_{ilt}} = \Big[\beta_{li}\Big(\frac{\lambda_i\varepsilon_t L_{iht}}{\chi-\rho+\psi+\eta_a}\Big)^{\beta_{ai}(1-\kappa_i)}\Big(\frac{\lambda_i\kappa_i\beta_{ai}p_i(1-\beta_{li}A_i^0)}{\beta_{li}p_z(r+\eta_a)}(l_{ilt}+\alpha_i l_{iht})\Big)^{\beta_{ai}\kappa_i}$$
$$\Big(\frac{(1-\beta_{li}A_i^0)\beta_{ki}p_i(l_{ilt}+\alpha_i l_{iht})}{\beta_{li}p_k(r+\eta_k)}\Big)^{\beta_{ki}}\varepsilon_t^{\beta_{li}}(l_{ilt}+\alpha_i l_{iht})^{\beta_{li}-1}\Big]^{\frac{1}{1-\beta_{ki}-\beta_{ai}\kappa_i}}$$

Since $L_{ih} = l_{ih}n_i$, $L_{il} = l_{il}n_i$, and $q(\theta_i)v_i = (\chi+\delta+\zeta)$, we get (46).

Proof of Proposition 1 First, suppose that there exists a balanced growth path. The stationarity conditions dictate that θ_{at} and θ_{mt} should be constant on the balanced growth path. Then, by construction, equation (52) should be satisfied on the balanced growth path. Second, suppose that the system of equations described in (52) has a solution of (θ_a, θ_m). Then, by invoking Lemma 1 through 4, we know that any pair of (θ_a, θ_m) can generate the solution of the model satisfying the equilibrium configuration (i) through (iv). Therefore, the solution of (52) solves for the balanced growth path.

Proof of Lemma 5 By the same reasoning as in the proof of Lemma 4, we get

$$k_{it} = \Big[\frac{(1-\beta_{li}A_i^0)\beta_{ki}p_{it}a_{it}^{\beta_{ai}}\varepsilon_t^{\beta_{li}}(l_{ilt}+\alpha_i l_{iht})^{\beta_{li}}}{(r+\eta_k)p_k}\Big]^{\frac{1}{1-\beta_{ki}}}. \tag{B33}$$

Since

$$\frac{\partial \pi_{it}}{\partial a_{it}} = \beta_{ai}p_i(1-\beta_{li}A_i^0)a_{it}^{\beta_{ai}}k_{it}^{\beta_{ki}}[\varepsilon_{it}(l_{ilt}+\alpha_i l_{iht})]^{\beta_{li}}\frac{1}{a_{it}}, \tag{B34}$$

we get

$$z_{it} = \varepsilon_t L_{iht}\Big[\frac{\lambda_i\kappa_i}{p_z}\int_t^\infty e^{-(r+\eta_a)(\tau-t)}\beta_{ai}p_i(1-\beta_{li}A_i^0)a_{i\tau}^{\beta_{ai}-1}k_{i\tau}^{\beta_{ki}}[\varepsilon_{i\tau}(l_{il\tau}+\alpha_i l_{ih\tau})]^{\beta_{li}}d\tau\Big]^{\frac{1}{1-\kappa_i}}$$

It completes the proof.